The roles of transparency in regime change: Striking when the iron's gone cold*

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August 2016

Abstract

How does freedom of information about an institution's resilience affect its stability? We study the *ex ante* impact of public information on regime change in a global game, accounting for uncertainty over what will be communicated. A fundamental tension exists in the ways public information impacts coordination. When the probability of regime change is already high, public information persuades agents into larger attacks. But under these conditions information targets attacks wastefully. For 'small' releases of public information, we characterize the overall implications of this trade-off for regime change. When the incumbent is ex ante weak, public information persuades agents to attack while simultaneously reducing their chances of success. By contrast, lower costs imply transparency negatively affects regime change only if the marginal productivity of attacks is sufficiently high.

Keywords: Public Information; Coordination; Regime Change

JEL Codes: C72, D62, D82, D83

*We thank Bob Evans, Sanjeev Goyal, Alexandre Kohlhas, Oliver Latham, Stephen Morris, Romans Pancs, Larry Samuelson, Balasz Szentes, Ansgar Walther, Peyton Young and seminar participants in Oxford, Cambridge and Vienna for helpful comments. The authors acknowledge financial support from the Economic and Social Research Council. The views expressed in this paper do not necessarily represent those of the IMF or IMF policy.

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1 Introduction

In recent years, access to information has played an important role as a catalyst in several major episodes of institutional instability. For instance much was made of the influence of media platforms such as Al-Jazeera and Twitter in the Arab Spring.[1] In the wake of the 2008 financial crisis, a *lack of transparency* has received considerable attention as a driver of market instability.[2] How does public information about an institution's resilience affect the stability of that institution? Can access to public information systematically bias group behaviour? Does information help groups to improve the efficacy of their coordination decisions?

In this paper we show that a fundamental tension exists in the roles of transparency as a persuasion and as a targeting device: when public information systematically (dis)incentivizes larger aggregate attacks, it conversely tends to distort (improve) the targeting or *efficacy* of attacks against a regime. In particular, transparency systematically incentivizes larger attacks when the probability of regime change is sufficiently high under an opaque system. By contrast, transparency improves attack efficacy when the probability of regime change under an opaque system is sufficiently low, pushing the incumbent over the edge when he is vulnerable to marginal attacks while reducing effort spent on inconsequential attacks. For small releases of public information we characterize the parameter conditions under which the trade-off between persuasion and targeting results in greater (lesser) regime change. Interestingly, we show that the targeting role can dominate the persuasion role, so that transparency incentivizes (disincentivize) attacks while simultaneously diminishing (increasing) the chances of successful regime change.

We develop these results in a standard global game of regime change.[3] A continuum of agents simultaneously choose whether to attack an incumbent institution, which is replaced with a known alternative if the aggregate attack exceeds an ex ante unknown threshold (the incumbent's *resilience*). Attacking is individually costly but agents receive a private payoff to being involved in a successful collective attack. Agents are privately and heterogeneously informed of the incumbent's resilience and may also observe a common signal before deciding to attack. Specifically we consider equilibrium in two scenarios: (i) opacity - where an

[1] For example, http://www.huffingtonpost.com/nehad-ismail/al-jazeeras-role-in-toppl_b_948247.html

[2] See, for example, http://www.brookings.edu/research/speeches/2011/09/06-financial-transparency-kohn and http://www.imf.org/external/pubs/ft/fandd/2013/12/brandao.htm

[3] Following Carlsson and van Damme [1993]'s seminal contribution, global games methods have been central to the study of public information in coordination games. Important contributions include, among others, Morris and Shin [1998], Corsetti et al. [2004], Goldstein and Pauzner [2005], de Mesquita [2010] and Amador and Weill [2010]. The original threshold model of political regime change is Granovetter [1978]. Lohmann [1994] develops a theory of regime change via herding. See Egorov et al. [2009] for a more recent contribution.

agent's information is limited to his private signal; (*ii*) transparency - where in addition individuals observe an informative public signal. We depart from the literature by explicitly distinguishing agents' priors and public information in a nonlinear coordination game, treating the latter as a random variable whose consequences depend on the realization observed. That is, we study the *ex ante impact* of transparency on the probability of regime change.[4] Seminal papers such as Morris and Shin [2002] and Angeletos and Pavan [2004], [2007a] and [2007b] study the impact of public information in beauty contest models, showing that it can cause *excess volatility* in outcomes.[5] By contrast our focus on incentive biases has no counterpart in these settings which rule out systematic impacts on average outcomes by assuming linear best responses.

To see the intuition for our main results, consider first the persuasion role of transparency. Under opacity agents attack if and only if their private signal is below a cut-off which we identify as the 'marginal agent'. Transparency increases the median attack size if the median public signal strictly incentivizes this marginal agent to attack. When does this occur? Suppose the probability of regime change under opacity is high relative to the cost of attack. Then the marginal agent in an opaque system must be more pessimistic than the average (otherwise he would strictly prefer to attack). Under transparency the median public signal reaffirms the prior belief, causing the marginal agent to place less weight on his own type and compressing his beliefs toward the population mean. As a result, he necessarily becomes less pessimistic and now strictly prefers to attack.

To study the targeting role, we consider how equilibrium outcomes vary across different realizations of public signals. Call public signals that incentivize large attacks signals of the incumbent's weakness and those that incentivize restraint signals of strength. Transparency improves the targeting of attacks when the incumbent's survival is more sensitive to marginal changes in attack size following a signal of weakness rather than a signal of strength. When does this occur? Suppose the probability of regime change under opacity is low. Since the incumbent survives with high probability, he is only likely to be vulnerable to marginal attacks when his resilience is lower than average. But since low signals express weakness, they are both highly correlated with vulnerable incumbents and simultaneously encourage larger attacks. Conversely signals of strength are unlikely to be observed when the incumbent is vulnerable at the margin. Thus, signals of strength save costly effort and have little impact on the regime's prospects while signals of weakness increase attack size when it matters.

Generally, the overall impact of transparency on regime change depends on the persuasion

[4]This is important since evaluations of transparency such as media freedom or central bank disclosures should acknowledge the consequences of all likely messages those institutions might send.

[5]Colombo et al. [2014] consider information acquisition in such settings.

and targeting roles but also a third channel which we refer to as *bandwagons*.[6] We derive a novel equation that allows us to characterize the conditions under which transparency increases regime change, for 'small' releases of information. We then use this equation to develop comparative statics in a class of models where bandwagon effects are small. We show that public information increases the probability of regime change when I's prior resilience is sufficiently high, despite persuading agents to attack in smaller numbers - the targeting role is more sensitive to I's resilience than the persuasion role. On the other hand, comparative statics in the costs of attacking are ambiguous, and depend on the returns to aggregate attacks. Transparency increases the chances of regime change when costs are large only if returns to participation are also high. Finally we show in a broad class of models conditions under which bandwagon effects are most important for outcomes.

In applications and extensions relevant to macroeconomics, finance and political economy, we use the decomposition to evaluate the overall impact of transparency on regime change. In each case we isolate either persuasion, targeting or bandwagons as the dominant channel and characterize the comparative statics of transparency. Studying the impact of transparency in these settings allows us to shed light on implicit assumptions on the informational structure which drive previous results in the literature.

Our work is closely related to a recent literature on transparency and the probability of regime change (see Iachan and Nenov [2015], Bouvard et al. [2015] and Szkup and Trevino [2015]).[7,8] To the best of our knowledge, ours is the first paper to provide a general analysis of the *ex ante* impact of unbiased public information on regime change. Interestingly our comparative statics results differ from models which study the effect of prior informativeness on regime change, where a more informative (lower variance) prior increases regime change if and only if the chances of success are already sufficiently high. The effect of prior informativeness is similar to our persuasion role. However, in our model public information also allows agents to target their aggregate attacks and this second effect can outweigh the first.

The remainder of the paper proceeds as follows. Section 2 presents the model. In section 3, we establish our main results on the competing roles of transparency for the probability of regime change. We bring these results together in section 4, where we study the overall effect of public information on regime change. Section 5 explores some applications and extensions of the model. Section 6 concludes. All proofs are relegated to Appendix A.

[6]Agents 'jump on the bandwagon' when signals of weakness have a larger impact on participation in attacks than signals of strength

[7]See also Chan and Chiu [2002], Bannier and Heinemann [2005], Metz [2002], Moreno and Takalo [2016].

[8]Edmond [2013] shows that additional media outlets can significantly damage the chances of regime change when the incumbent can manipulate the information transmitted. As we show, manipulation is not necessary for public information to protect regimes.

2 A Model of Regime Change

Agents face a coordination problem when assessing whether to attack a regime: they will only be successful if enough of them join in. We take the standard approach of modeling this as a global game.

2.1 The Model

There is a continuum of agents, uniformly distributed on $[0,1]$ with measure one. Agents are indexed by i, their position on $[0,1]$. Each agent i simultaneously chooses whether to attack an incumbent institution, I $(a(i)=1)$ or refrain from attacking $(a(i)=0)$. The aggregate attack is $A=\int_0^1 a(i)\,di$. An aggregate attack is successful if it *overthrows* I. Otherwise I *survives.*

2.1.1 Individual Payoffs

Our payoff structure closely follows several standard models of regime change.[9] Participating in an attack imposes a personal cost of c on the agent. In the event that regime change is successful any participating agent receives a payoff of $1+q$, $q\in\mathbb{R}$, from overthrowing I: First, he gets a normalized private payoff of 1 from participation. Second, we allow for externalities by giving agents a public payoff q. If the agent decides not to participate, he gets q if I is overthrown and 0 otherwise.[10]

To make the problem interesting, we assume throughout that $0<c<1$. Then an agent is incentivized to attack if he expects regime change to be successful, but would rather refrain if he believes it will fail.

In places, we will be interested in the behavior of an incumbent who can control the flow of public information to the agents. We simply assume that I prefers to survive over being overthrown. We normalize these payoffs to 1 and 0, respectively.

2.1.2 Returns from Coordination

Given an aggregate attack, A, I is overthrown if $\theta\le f(A)$ where $f:[0,1]\to\mathbb{R}$ is an increasing, C^2 function, with $|f'(A)|\le F_1$, $|f''(A)|\le F_2$ for $F_1,F_2\in\mathbb{R}_{++}$.[11] We refer to θ as I's

[9] See for instance, Angeletos et al. [2006] and Edmond [2013]. Iachan and Nenov [2015] consider a richer payoff structure but a more limited information structure.

[10] The interpretation of the costs and benefits depends on the exact application. In the political regime change context c could be the expected cost of death/punishment, the private payoff the benefit of being part of the future elite (see Edmond [2013]) and q the social benefit (cost) of changing leader.

[11] We assume bounded second derivatives for ease of exposition only. In particular, our main result, Proposition 4, continues to hold so long as f'' does not approach infinity too fast as $A\to 0,1$.

resilience - a higher value of θ implies that a larger attack is required to overthrow the incumbent. The function f models the productivity of aggregate attacks (see Iachan and Nenov [2015] for a similar assumption). The case most used in the literature is $f(A) = A$.

2.1.3 Information

Private Information

Agents share the common prior that θ is normally distributed: $\theta \sim N\left(\bar{\theta},\, \sigma_\theta^2\right)$, where $\sigma_\theta^2 > 0$. Each agent i receives a private signal (his 'type'), $x(i) = \theta + \epsilon(i)$ where $\epsilon(i) \sim N(0,\, \sigma_\epsilon^2)$ is *i.i.d.* across i. We drop the i notation and write $x(i) = x$ where clear, but private signals are always conditionally independent across agents. By standard properties of the joint normal distribution, agent i's beliefs about the state conditional on type x are:

$$\theta \mid x \sim N\left(\omega_\emptyset x + (1 - \omega_\emptyset)\,\bar{\theta},\, \omega_\emptyset \sigma_\epsilon^2\right) \tag{1}$$

where $\omega_\emptyset = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\epsilon^2}$ is the weight agent i places on his type in his posterior expectation of θ.

Public Information

We consider two alternative informational environments. Under *opacity*, each agent i observes only his own type. Under *transparency*, agents commonly observe a public signal $y = \theta + \eta$ where $\eta \sim N\left(0,\, \sigma_\eta^2\right)$, uncorrelated with $\epsilon(i)$, $\forall i$.

Define $Z := \{\emptyset\} \bigcup \mathbb{R}$, where $z = y$ if $z \in Z \setminus \{\emptyset\}$. A realized public signal $z \in Z$ induces a common belief $\Pr(\theta \mid z)$ over I's resilience. Given standard properties of the bivariate normal distribution, we can express this belief compactly as

$$\theta \mid z \sim N\left(\mu_z, \sigma_z^2\right)$$

where $\sigma_\emptyset^2 = \sigma_\theta^2$, $\sigma_y^2 := \frac{\sigma_\theta^2 \sigma_\eta^2}{\sigma_\theta^2 + \sigma_\eta^2}$, $\mu_\emptyset = \overline{\theta}$ and $\mu_y = \left(\frac{\sigma_\eta^2}{\sigma_\theta^2 + \sigma_\eta^2}\right)\overline{\theta} + \left(\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\eta^2}\right) y$.

As is typical in global games, we assume that both the prior and public information are not 'too' precise relative to private information in order to ensure equilibrium uniqueness. Specifically, we assume

$$\frac{\sigma_y^2}{\sigma_\epsilon} > F\,(2\pi)^{-\frac{1}{2}} \tag{2}$$

2.1.4 Strategies

A pure strategy for agent i is a function $\alpha_i : \mathbb{R} \times Z \to \{0,\, 1\}$, where $\alpha_i(x,\, \emptyset)$ is the action taken by type x under opacity and $\alpha_i(x,\, y)$ the action that type x takes if he also observes public signal y. As is common in global games, symmetric cut-off strategies turn out to be central to our analysis. Formally,

Definition. In a symmetric cut-off strategy, $\alpha_i(x,z) = 1$ iff $x \leq x_z$, for some function $x_z : Z \to \mathbb{R}$. We call x_z the z-marginal type.

Throughout the rest of the paper, we will identify a cut-off strategy with its marginal type, x_z.

Suppose I will be overthrown if $\theta \in \tilde{\theta}$, for some $\tilde{\theta} \in \mathbb{R}$. A best response for agent i with type x and public signal z sets $a_i = 1$ iff[12]

$$\Pr\left(\theta \leq \tilde{\theta} \mid x,\, z\right) \geq c \tag{3}$$

In keeping with the literature, our model exhibits two-sided limit dominance. First, it is easy to see that there exists an $\overline{x}$ large enough that $\Pr(\theta < f(1) \mid x) \leq c$ for all types $x \geq \overline{x}$ since $f(1)$ is finite and, from (1), $\Pr(\theta < f(1) \mid x)$ limits to 0 as $x \to \infty$. Consequently there will always exist types high enough who have a dominant strategy to refrain. Likewise, there always exist types low enough that attacking is strictly dominant.

2.2 Continuation Equilibrium

A Perfect Bayesian Equilibrium of the model consists of strategies $\alpha_i : \mathbb{R} \times Z \to \{0,1\}$ and an aggregate attack function $A : \mathbb{R} \to [0,1]$ such that *(i)* $a_i = 1$ iff $\Pr(\theta \leq f(A(\theta)) \mid x,\, z) \geq c$, *(ii)* $A(\theta) = \mathbb{E}[\alpha_i(x,z) \mid \theta]$.

Conditional on any common signal $z \in Z$, the analysis of equilibrium reduces to a standard global game. Given assumption (2) and two-sided limit dominance, it is well known that there is a unique continuation equilibrium determined by two threshold rules: a symmetric cut-off strategy such that an agent attacks iff $x \leq x_z^*$ and a threshold θ_z^* such that the incumbent is overthrown iff $\theta \leq \theta_z^*$.[13] These thresholds are determined as the solution to the system:

$$\theta_z = f\left(\Phi\left(\frac{x_z - \theta_z}{\sigma_\epsilon}\right)\right) \tag{4}$$

$$c = \Phi\left(\frac{\theta_z - \omega_z x_z - (1-\omega_z)\,\mu_z}{\sigma_\epsilon \sqrt{\omega_z}}\right) \tag{5}$$

where Φ is the standard normal distribution function and $\omega_z = \frac{\sigma_z^2}{\sigma_z^2 + \sigma_\epsilon^2}$.

[12]We assume that whenever an agent is indifferent, he will attack. This does not affect any of the results, because such individuals occur with measure 0 in any equilibrium.

[13]We use * notation to distinguish equilibrium quantities from their out-of-equilibrium counterparts.

The first equation characterizes the production function for successfully overthrowing I. If all agents attack when their type is below x_z, then the aggregate attack size given I's resilience θ is just $A_z(\theta) = \Phi\left(\frac{x_z - \theta_z}{\sigma_\epsilon}\right)$. Note that this is decreasing in θ. Equation (4) thus uniquely determines the least resilient regime type that can survive such attacks. As we make use of it in section 4 and in the proofs, we define the solution curve that solves (4) as the function $\theta_P(x)$.

The second equation is a supply function for aggregate attacks. Suppose all agents expect I will be overthrown if $\theta \leq \theta_z$. Then an individual agent attacks if $\Pr(\theta \leq \theta_z \mid x, z) \geq c$. Equation (5) identifies the marginal agent, x_z, who is just indifferent between attacking and refraining and therefore pins down the set of types $x \leq x_z$ who are willing to attack if they expect I will be overthrown whenever it is weaker than a threshold θ_z. At an equilibrium, (4) and (5) must be mutually consistent. Under assumption (2), it is well-known that there is a unique (x_z^*, θ_z^*) solving these equations, $\forall z \in Z$. Armed with this characterization of the continuation equilibrium, next we analyze the roles of transparency in regime change games.

3 The Roles of Transparency

We now turn to our main questions: how public information affects *(i)* incentives to attack at the margin and *(ii)* the efficacy of collective attacks against the incumbent. Since it is costly for individuals to attack, it is interesting in its own right to understand whether public information helps to achieve regime change by encouraging attacks or by targeting them more effectively. Moreover, we find that these two channels are surprisingly often (but not always) in conflict: when public information is likely to increase aggregate attacks, larger attacks also tend to be poorly targeted.

We assume that I can commit to either provide the random variable Y to all agents publicly (*transparency*) or withhold it (*opacity*), before observing the particular realization, y.[14] While it is often difficult for political regimes to credibly disclose unbiased information, commitment is a relevant assumption in several instances. When the government allows new communication technologies (explicitly or implicitly, by not enforcing restrictions), it cannot always control the content shared via those platforms.[15] In a banking context, some

[14] Alternatively, we can think of public information as outside of the incumbent's control. Our results generally provide a positive comparison of equilibria with and without public signals.

[15] Tufekci and Wilson [2012] make this point in the context of social media in the Arab Spring. While China has significant capacity to monitor social media (King et al. [2013]), it does not censor all criticism of the state and censorship is reactive.

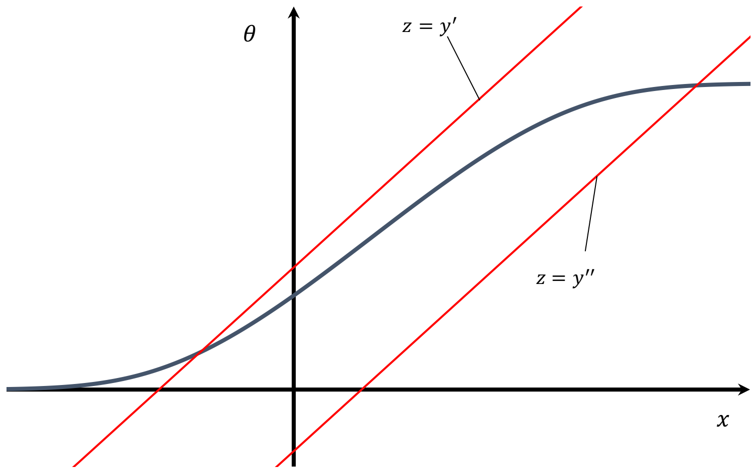


Figure 1: Blue curve represents solutions to equation (4); Red lines display solutions to (5), for signal realisations $z = y', y''$, where $y'' < y'$. In each case, equilibrium is uniequly determined by the intersection of the two curves.

regulators have successfully committed to providing regular stress tests.[16,17]

To study the roles of transparency on regime change, we briefly present some comparative statics properties of x^*_z and θ^*_z. These help build intuition and are useful for our main analysis below:

Lemma 1. Any continuation equilibrium satisfies the following:

1. For any $z \in Z$, the unique pair (x^*_z, θ^*_z) solving (4) - (5) is continuous and strictly decreasing in c, $\overline{\theta}$ and $z \in \mathbb{R}$.

2. $\theta^*_z \geq x^*_z$ iff $x^*_z \leq f\left(\frac{1}{2}\right)$.

3. Let $z = \emptyset$. For any (x', θ') solving (4), there is a continuous, decreasing curve $c\left(\overline{\theta} \mid \theta'\right)$, $c \in (0,1)$, such that (x', θ') is an equilibrium under opacity for parameters $\left(\overline{\theta}, c\left(\overline{\theta} \mid \theta'\right)\right)$.

Lemma 1.1 tells us that the equilibrium cut-off x^*_z strictly decreases in c, $\overline{\theta}$ and z, for $z \neq \emptyset$. This is intuitive: an increase in the cost of attack or a publicly observed increase in regime's expected resilience, μ_z, reduces the net benefit of attacking for all agents. Since fewer types attack, naturally the threshold θ^*_z falls too. Figure 1 illustrates equilibrium determination for two possible realizations of the public signal, $y' > y$.[18]

[16] See Bouvard et al. [2015]. Freixas and Laux [2011] and Schuermann [2014] argue that U.S. policymakers were better able to commit their European counterparts during the crisis.

[17] In the Online Appendix, we consider the effect of commitment problems on equilibrium disclosures by the incumbent.

[18] Note that that the Lemma does not directly compare $x^*_\emptyset$ to x^*_y - we deal with that in the next section.

Lemma 1.2 relates the threshold θ_z^* directly to x_z^*. As we will see below, this property turns out to be important for understanding how the persuasion and targeting roles of transparency interact. To gain intuition, consider equation (4) and a cut-off strategy $x^{'}$, such that I is overthrown if and only if $\theta \leq \theta'$. A higher cut-off $x'' > x'$ induces larger aggregate attacks and therefore a higher threshold $\theta'' > \theta'$. Since f is strictly increasing, inspection of (4) reveals that the difference $x'' - \theta''$ must be strictly greater than $x' - \theta'$. Since equation (4) does not explicitly depend on c or μ_z, the same logic must apply to comparing equilibrium thresholds across parameters. Specifically, equation (4) shows that a necessary condition for an equilibrium with $x_z^* = \theta_z^*$ is $\theta_z^* = f\left(\frac{1}{2}\right)$. Finally, it is easy to verify parameters for which this is indeed an equilibrium.[19]

Lemma 1.3 tells us that $\overline{\theta}$ and c can always be traded-off to determine any opaque equilibrium. Given any (x', θ') solving (4) and some $\overline{\theta}$, we can always find a unique c solving (5). Since the RHS of (4) decreases continuously in $\overline{\theta}$, so too must $c\left(\overline{\theta}\right)$. While simple, this point is important for establishing the comparative statics results in sections 3.1 and 3.2.

3.1 Persuasion

In this section, we ask: can transparency systematically incentivize agents into larger attacks? To answer this question, we compare the median attack size under opacity to the median attack size under transparency, accounting in the latter case for the randomness introduced by the public signal y on equilibrium behavior. More specifically, we characterize necessary and sufficient conditions under which $\Pr_{\theta,y}\left[A_y(\theta) \geq A_\emptyset(\theta)\right] > \frac{1}{2}$, where $\Pr_{\theta,y}$ is the measure induced by the joint RV (θ, y).

We define a 'neutral signal', y_n, as the realization of the public signal that yields the same equilibrium cut-offs as under opacity. That is, y_n satisfies

$$\left(x_{y_n}^*, \theta_{y_n}^*\right) = \left(x_\emptyset^*, \theta_\emptyset^*\right)$$

Lemma 1 implies y_n is always well-defined. A characterization of the neutral signal turns out to be important to answer our main question, as the following Lemma makes clear:

Lemma 2. $\Pr_{\theta,y}\left[A_y(\theta) \geq A_\emptyset(\theta)\right] \geq \frac{1}{2}$ if and only if $y_n \geq \overline{\theta}$, with equality iff $y_n = \overline{\theta}$.

Lemma 2 tells us that the median size of equilibrium attacks increases if and only if the neutral signal lies in the right tail of the prior distribution of y. If $y_n > \overline{\theta}$, the neutral signal is above the mean (and median) of the ex ante distribution of y and by Lemma 1.1,

[19] For instance, $c = \frac{1}{2}$, $\overline{\theta} = f\left(\frac{1}{2}\right)$, $z = \emptyset$. Lemma 1.2 then implies that, given any values of any two of these parameters, we can always find a value for the third such that $x^* = \theta^* = f\left(\frac{1}{2}\right)$ remains an equilibrium.

transparency must lead to larger attacks with a probability of more than $\frac{1}{2}$. However this simple Lemma offers no guidance about when y_n is likely to exceed $\overline{\theta}$. The main result of this section characterizes the answer to this question as a function of the equilibrium threshold under opacity (and therefore of the underlying parameters, c, $\overline{\theta}$):

Proposition 1. There exists a continuous function $\lambda : [f(0), f(1)] \to \mathbb{R}$ such that

$$\Pr_{\theta,y}[A_y(\theta) \geq A_\emptyset(\theta)] > \frac{1}{2} \iff \bar{\theta} \leq \theta^*_\emptyset + \lambda(\theta^*_\emptyset)$$

Moreover, $\lambda\left(\theta^*_\emptyset\right) \leq 0$ iff $\theta^*_\emptyset \leq f\left(\frac{1}{2}\right)$.

Proposition 1 gives necessary and sufficient conditions under which transparency increases the median attack size. It shows that transparency increases the median attack size whenever the prior mean $\overline{\theta}$ is sufficiently low relative to the equilibrium threshold under opacity, $\theta^*_\emptyset$, and vice versa. Since

$$\Pr(\theta \leq \theta^*_\emptyset) = \Phi\left(\frac{\theta^*_\emptyset - \overline{\theta}}{\sigma_\theta}\right)$$

the Proposition equivalently states that public information increases the median attack size when the probability of regime change under opacity is already sufficiently high. The function $\lambda\left(\theta^*_\emptyset\right)$ determines just how low $\overline{\theta}$ must be relative to $\theta^*_\emptyset$ before transparency increases median attacks. When equilibrium attacks are relatively small ($f^{-1}\left(\theta^*_\emptyset\right) < \frac{1}{2}$), the probability of regime change under opacity must be strictly greater than $\frac{1}{2}$ before transparency will increase median attacks. Conversely, when $\theta^*_\emptyset \geq f\left(\frac{1}{2}\right)$ transparency can systematically increase attacks even if $\Pr\left(\theta \leq \theta^*_\emptyset\right) < \frac{1}{2}$.

To see the intuition, suppose that equilibrium under opacity $\left(x^*_\emptyset, \theta^*_\emptyset\right)$ is such that $\theta^*_\emptyset > f\left(\frac{1}{2}\right)$. For ease of exposition suppose first that $\overline{\theta} = \theta^*_\emptyset$, where the ex ante probability of regime change under opacity is exactly $\frac{1}{2}$.[20] The aggregate attack in such an equilibrium is $A_\emptyset(\theta) = \Phi\left(\frac{x^*_\emptyset - \theta}{\sigma_\epsilon}\right)$.

Now suppose that I chooses transparency. From Lemma 2, public information increases aggregate attacks with probability greater than $\frac{1}{2}$ iff

$$A_{\overline{\theta}}(\theta) > A_\emptyset(\theta)$$

or equivalently, if $x^*_{\overline{\theta}} > x^*_\emptyset$. Thus, transparency systematically increases attacks if even a signal $y = \overline{\theta}$ strictly incentivizes the $\emptyset$-marginal agent to attack at the margin.

When $\theta^*_\emptyset > f\left(\frac{1}{2}\right)$, we know from Lemma 2.2 that the $\emptyset$-marginal signal must satisfy $x^*_\emptyset > \theta^*_\emptyset$. But since $\theta^*_\emptyset = \overline{\theta}$, the $\emptyset$-marginal agent is a *relative pessimist*: he believes that I is

[20] Given Lemma 1.3, we can always find a c to support this as an equilibrium.

likely more resilient than the average member of the population since

$$\omega x_\emptyset^* + (1-\omega)\,\overline{\theta} > \overline{\theta}$$

By reducing the weight that the pessimistic $\emptyset$-marginal agent puts on his own private signal ($\omega_y < \omega_\emptyset$), even a signal $y = \overline{\theta}$ that simply reaffirms the prior can strictly induce the $\emptyset$-marginal agent into attacking. Of course, as we mentioned above public signals also have the effect of reducing the agent's posterior variance. We show in the Appendix that the effect of transparency on the mean belief dominates.

When $\overline{\theta} < \theta_\emptyset^*$, this persuasion role of transparency is intuitively even more pronounced since the difference between $x_\emptyset^*$ and $\overline{\theta}$ is larger. Furthermore, recalling from Lemma 1.2 that any opaque equilibrium threshold $\theta_\emptyset^*$ can be supported by curve $\left(\overline{\theta}, c\left(\overline{\theta} \mid \theta_\emptyset^*\right)\right)$, a compensating increase in c (for a given $\theta_\emptyset^*$) makes the variance effect also more likely to incentivize attacks.

While Proposition 1 is nominally stated in terms of the endogenous $\theta_\emptyset^*$, it can easily be recast as a function of $\overline{\theta}$ and c using the curve $c\left(\overline{\theta} \mid \theta_\emptyset^*\right)$. Then, Proposition 1 states that transparency increases median attacks when $\overline{\theta}$ is sufficiently low relative to c. Indeed, the proof in the Appendix shows that as we decrease $\overline{\theta}$ along curve $c\left(\overline{\theta} \mid \theta_\emptyset^*\right)$, there is a threshold below which $y_n > \overline{\theta}$. Finally, notice that the Proposition is able to shed light on what happens when only $\overline{\theta}$ changes. Consider successively reducing $\overline{\theta}$, holding c constant. In that case, Lemma 1.1 tells us that $\theta_\emptyset^*$ eventually increases above $f\left(\frac{1}{2}\right)$ and $\overline{\theta}$ itself. Thus, for low enough $\overline{\theta}$, it continues to be the case that transparency will incentivize larger attacks.

Spontaneous Attacks

We might find it surprising that transparency increases equilibrium attacks only when regime change is already sufficiently likely under opacity. After all, information typically persuades an individual to attack only if it was not already optimal for him to do so under opacity. This highlights a tension between the ways in which individuals and groups respond to information. In particular, we show here that even when transparency dissuades the majority of agents from attacking, the median size of aggregate attacks can increase.

Following immediately from Proposition 1, we have:

Corollary 1. Suppose that $\bar{\theta} \leq \theta_\emptyset^* + \lambda\left(\theta_\emptyset^*\right)$. If in addition $x_\emptyset^* \geq \overline{\theta}$, transparency induces *spontaneous attacks.* That is,

$$\Pr_{\theta,y}\left[A_y(\theta) \geq A_\emptyset(\theta)\right] > \frac{1}{2}$$

while

$$\Pr_{x}\left[\mathbb{E}\left[\alpha^*(x,y) \mid x\right] \leq \alpha^*(x,\emptyset)\right] > \frac{1}{2}$$

where $\alpha^* : \mathbb{R} \times Z \to \{0,1\}$ is the equilibrium symmetric cut-off strategy.[21]

Corollary 1 gives conditions under which the majority of agents become *less* inclined to attack when we introduce public information and yet aggregate attacks typically increase. $\Pr_x [\mathbb{E}[\alpha^*(x,y) \mid x] \leq \alpha^*(x,\emptyset)]$ is the share of agents who expect to attack less under transparency.[22] Intuitively, when $x^*_\emptyset \geq \overline{\theta}$ the marginal agent is more pessimistic than the majority, who can only be disincentivized from attacking at the margin.

3.2 Targeting

Having seen how transparency can systematically incentivize attacks we now consider whether those attacks get directed effectively, helping agents to push fragile regimes over the edge rather than wasting large attacks on regimes weak enough to fall without any extra help.

Formally, for any γ, $k \in \mathbb{R}_+$, we say that transparency enhances the efficacy of attacks if

$$\Pr_\theta [\theta \in [\theta^*_\emptyset, \theta^*_\emptyset + k] \mid y_n - \gamma] > \Pr_\theta [\theta \in [\theta^*_\emptyset - k, \theta^*_\emptyset] \mid y_n + \gamma] \tag{6}$$

where $\Pr_\theta [\cdot \mid y]$ is the measure induced by the RV $\theta \mid y$. The left-hand side of (6) is the probability that I's true resilience is just above $\theta^*_\emptyset$, conditional on observing a public signal $y_n - \gamma$, or I's *vulnerability to increased attacks at the margin* when $y_n - \gamma$ is observed. Note that since $y_n - \gamma < y_n \implies x^*_{y_n-\gamma} > x^*_\emptyset$, such a public signal does in fact incentivize larger attacks than under opacity. Similarly, the right-hand side measures I's marginal vulnerability for signals $y_n + \gamma$ that disincentivize the agents from attacking. Therefore, (6) tells us that transparency enhances the efficacy of attacks if it induces larger attacks when I is most vulnerable.

Before proceeding with the analysis, we briefly elaborate on some details of our definition in (6). First, we choose to compare I's vulnerability for signals $y_n - \gamma$, $y_n + \gamma$ that are equidistant from the neutral signal. This ensures that we fairly compare signals on a fundamental basis.[23] Moreover, our definition of vulnerability compares intervals of (arbitrary) equal length, k. This allows us to isolate the relationship between I's vulnerability and the *direction* in which y shifts aggregate attacks at the margin, independent of the dynamics of coordination. In particular, we do not claim in (6) that fundamentally opposite signals

[21] For ease of exposition, we assume for the rest of this section that $\alpha^*(x^*_\emptyset, \emptyset) = 0$. Since type $x^*_\emptyset$ has 0 mass and is indifferent between attacking and refraining when $z = \emptyset$, this profile remains an equilibrium and has no effect on aggregate outcomes.

[22] Recalling $\alpha^*(x,\emptyset) = 1$ iff $x \leq x^*_\emptyset$, this is also the share of agents who believe transparency can *only* disincentivize them from attacking.

[23] Indeed, recalling from section 2 that $\theta \mid y \sim N(\mu_y, \sigma^2_y)$, signals $y_n + \gamma$ and $y_n - \gamma$ simply shift the distribution of θ by an equal and opposite amount relative to the neutral signal.

$y_n - \gamma$, $y_n + \gamma$ also have equal and opposite effects on the equilibrium threshold, θ_y^*. Indeed, Figure 1 illustrates that this is usually not the case. We deal with nonlinearities introduced by the dynamics of coordination and the function f in section 3.3.

Again, the neutral signal playds a central role for determining the efficacy of attacks:

Lemma 3. $\Pr_\theta\left[\theta \in \left[\theta_\emptyset^*, \theta_\emptyset^* + k\right] \mid y_n - \gamma\right] \geq \Pr_\theta\left[\theta \in \left[\theta_\emptyset^* - k, \theta_\emptyset^*\right] \mid y_n + \gamma\right]$, $\forall \gamma, k \in \mathbb{R}_+$, if and only if $\mu_{y_n} \geq \theta_\emptyset^*$, with equality iff $\mu_{y_n} = \theta_\emptyset^*$.

Lemma 3 states that transparency enhances the efficacy of attacks if and only if a public realization y_n signals that I's resilience is expected to exceed $\theta_\emptyset^*$. Intuitively, if $\mu_{y_n} > \theta_\emptyset^*$ then the realization $y_n + \gamma$ would inform us that a value of θ close to $\theta_\emptyset^*$ is an even more extreme left tail event than under realization y_n, with a corresponding low probability. By contrast, $\theta_\emptyset^*$ is not as far in the tail of the conditional distribution $\theta \mid y_n - \gamma$, implying that transparency increases the efficacy of attacks.

While Lemma 3 provides a test for understanding when transparency is efficacy-enhancing it sheds no light on when the endogenous quantity y_n is likely to pass that test. The next Proposition answers this question:

Proposition 2. There exists a continuous function $\delta : [f(0), f(1)] \to \mathbb{R}$ such that

$$\Pr_\theta\left[\theta \in [\theta_\emptyset^*, \theta_\emptyset^* + k] \mid y_n - \gamma\right] > \Pr_\theta\left[\theta \in [\theta_\emptyset^* - k, \theta_\emptyset^*] \mid y_n + \gamma\right] \iff \bar{\theta} \geq \theta_\emptyset^* + \delta\left(\theta_\emptyset^*\right)$$

for all k, $\gamma \in \mathbb{R}_+$. Moreover, $\delta\left(\theta_\emptyset^*\right) \geq 0$ iff $\theta_\emptyset^* \leq f\left(\frac{1}{2}\right)$.

Proposition 2 gives necessary and sufficient conditions under which transparency enhances the efficacy of attacks. Transparency is efficacy-enhancing whenever the prior mean $\overline{\theta}$ is sufficiently high relative to the equilibrium threshold under opacity, $\theta_\emptyset^*$. Conversely, transparency reduces median attacks when $\overline{\theta}$ is sufficiently high. The function δ characterizes just how high $\overline{\theta}$ must be before transparency improves the efficacy of attacks. In particular, notice that if $\theta_\emptyset^* \geq f\left(\frac{1}{2}\right)$ transparency strictly improves attack efficacy even when $\Pr\left(\theta \leq \theta_\emptyset^*\right) = \frac{1}{2}$. Again, we can interpret the Proposition 2 in terms of the probability of regime change under opacity. In particular, transparency better targets attacks when the probability of regime change under opacity is sufficiently low.

To see the intuition, suppose that $\bar{\theta}$ is high relative to $\theta_\emptyset^*$. Given equation (5) evaluated at $z = \emptyset$, this can only be maintained as an equilibrium if the cost of attacking is sufficiently low to just incentivize type $x_\emptyset^*$ to attack. But comparing type $x_\emptyset^*$'s incentives under opacity and under the neutral signal y_n,

$$\Phi\left(\frac{\theta_\emptyset^* - \omega_y x_\emptyset^* - (1 - \omega_y)\,\mu_{y_n}}{\sigma_\epsilon \sqrt{\omega_y}}\right) - c = 0$$

we can see that if $\bar{\theta}$ is large enough relative to $\theta^*_\emptyset$, then μ_{y_n} must also be relatively large for the $\emptyset$-marginal agent to remain indifferent when given neutral news. In other words, if he believes there is a low chance of success under opacity (because $\theta^*_\emptyset < \bar{\theta}$), he must also believe there is a low chance of success under the neutral signal. Otherwise, he would be strictly incentivized to attack (recall c is small). Applying Lemma 3, transparency must therefore be efficacy-enhancing.

Propositions 1 and 2 highlight a fundamental tension in the ability of transparency to act as a persuasion and a targeting device. When regime change is already likely even under opacity, transparency systematically incentivizes agents into larger aggregate attacks (Proposition 1) but these large attacks are targeted wastefully on regimes that woud be too weak to survive even under opacity (Proposition 2). We provide a more detailed analysis of this trade-off in section 3.3.

3.3 Bandwagons

As mentioned in section 3.2 equilibrium outcomes typically vary nonlinearly in y, which is not captured in either the persuasion or targeting roles of transparency. We now consider how much the attack size changes in response to different signals. This is particularly important when individuals face a coordination motive since strategic complementarities make participation in attacks contagious. We show that transparency can affect the chances of regime change because the strength of group attacks, $f(A_y(\theta))$, changes asymmetrically in response to fundamentally equivalent signals.[24]

We say that two fundamentally opposite public signals $y_n - \gamma$, $y_n + \gamma$ induce agents to *jump on (off) the bandwagon* if

$$x^*_{y_n-\gamma} - x^*_\emptyset > (<)\, x^*_\emptyset - x^*_{y_n+\gamma} \tag{7}$$

(7) is a natural definition: if a public signal $y_n - \gamma$ incentivizes strictly more types of agent to attack in equilibrium than its fundamental opposite $y_n + \gamma$ discourages attacks, there must be types who attack for reasons over and above their fundamental beliefs in I's resilience. Indeed, it turns out that if signals $y_n + \gamma$ and $y_n - \gamma$ had equal and opposite effects on the threshold θ^*, then we would have $x^*_{y_n-\gamma} - x^*_\emptyset = x^*_\emptyset - x^*_{y_n+\gamma}$. More generally:

Lemma 4. $x^*_{y_n-\gamma} - x^*_\emptyset \geq (<)\, x^*_\emptyset - x^*_{y_n+\gamma}$ if and only if

$$\theta^*_{y_n-\gamma} - \theta^*_\emptyset \geq (<)\, \theta^*_\emptyset - \theta^*_{y_n+\gamma} \tag{8}$$

[24] Morris and Shin [2000] characterize a publicity multiplier in these settings. They consider a linear approximation of equilibrium in y and so do not evaluate the kind of nonlinearities we look at here.

Lemma 4 tells us that transparency induces agents to jump on the bandwagon if and only if signals of weakness have outsized impacts on equilibrium regime change. To see this, let $\Delta x := x^*_\emptyset - x^*_{y_n+\gamma}$, $\Delta\theta_{y_n+\gamma} := \theta^*_\emptyset - \theta^*_{y_n+\gamma}$ and suppose that a threshold strategy to attack iff $x \leq x^*_\emptyset + \Delta x$ overthows I for $\theta \leq \theta^*_\emptyset + \Delta\theta$, where $\Delta\theta > \Delta\theta_{y_n+\gamma}$. In this scenario, an incremental Δx of types attacking has a larger impact on the threshold θ^* than does an incremental Δx refraining. Noting that $\Delta\theta_{y_n+\gamma} = \omega_y \Delta x + (1-\omega_y)\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\eta}\gamma$ (since (5) must hold for all y), it is easy to verify that beliefs satisfy

$$\Pr\left(\theta \leq \theta^*_\emptyset + \Delta\theta \mid x^*_\emptyset + \Delta x, y_n - \gamma\right) > \Pr\left(\theta \leq \theta^*_\emptyset - \Delta\theta_{y_n+\gamma} \mid x^*_\emptyset - \Delta x, y_n + \gamma\right) = c \qquad (9)$$

(9) tells us if a signal $y_n - \gamma$ incentivized Δx more types to attack, then type $x^*_\emptyset + \Delta x$ would strictly prefer to attack. Thus, more types are incentivized to jump on the bandwagon, providing an extra indirect effect that further boosts the probability of regime change such that $\theta^*_{y_n-\gamma} - \theta^*_\emptyset > \Delta\theta$.

To gain intuition for when agents will jump on the bandwagon, we establish comparative statics for the case $f(A) = A$:

Proposition 3. Suppose $f(A) = A$. $\theta^*_{y_n-\gamma} - \theta^*_\emptyset \geq \theta^*_\emptyset - \theta^*_{y_n+\gamma}$ if and only if

$$\theta^*_\emptyset \leq \frac{1}{2}$$

with equality iff $\theta^*_\emptyset = \frac{1}{2}$.

When $f(A) = A$, transparency induces agents to jump on the bandwagon when the threshold $\theta^*_\emptyset$ is relatively small. Recalling Lemma 1, under these conditions $x^*_\emptyset$ lies in the left tail of the type distribution at the marginal state $\theta^*_\emptyset$. Given types are ex ante normally distributed, a public signal $y_n - \gamma$ must therefore influence a larger mass of agents (those in $\left[x^*_\emptyset, x^*_\emptyset + \Delta x\right]$, for some $\Delta x \in \mathbb{R}_+$) than does $y_n + \gamma$ (those in $\left[x^*_\emptyset - \Delta x, x^*_\emptyset\right]$) at the margin. Of course this encourages more types to jump on the bandwagon for such signals.

4 The Equilibrium Impact of Transparency

We now turn to the overall impact of transparency on regime change. Our main result, Proposition 4, evaluates the relative contributions of persuasion, targeting and bandwagons to I's survival/removal in general, for large noise σ^2_η. We illustrate that the persuasion and targeting roles are informational effects, whose signs only depend on the relationship between the marginal agent's beliefs and the true fundamentals of the economy. By contrast the importance of bandwagons is driven by asymmetries in coordination dynamics across signals of I's strength/weakness, as captured by nonlinearities in the curve $\theta_P(x)$, the 'production function' for regime change. Since the drivers of bandwagons are somewhat orthogonal to those for persuasion and targeting, we then use Proposition 4 to make comparative statics predictions in two classes of problem. First, we consider environments where bandwagon effects are small for almost all parameters (piecewise linear $\theta_P(x)$). We then present results for more general $\theta_P(x)$.

To study the ex ante effect of public information on regime change, define

$$T(\rho) := \Pr_{\theta,y}\left(\theta \leq \theta^\star_y\right) - \Pr_{\theta}\left(\theta \leq \theta^\star_\emptyset\right) \tag{10}$$

where $\rho = (\sigma^2_\eta, \sigma^2_\theta, \sigma^2_\epsilon, \overline{\theta}, c, f)$ is the collection of model parameters. We show in the Appendix that $T(\rho)$ can be decomposed into three terms, $T_\mathcal{I}$, $T_\mathcal{T}$, and $T_\mathcal{B}$, which respectively represent the persuasion, targeting and bandwagon roles of transparency:

Lemma 5. $T(\rho) \equiv T_\mathcal{I}(\rho) + T_\mathcal{T}(\rho) + T_\mathcal{B}(\rho)$

$T_\mathcal{I}$ can be thought of as the hypothetical effect of transparency if signals $y_n \pm \gamma$ were to have an equal and opposite effect on I's survival, and therefore reflects the relative likelihood of such signals. $T_\mathcal{T}$ is an average over I's net resilience for signals $y_n - \gamma$ over $y_n + \gamma$. Finally, $T_\mathcal{B}$ is an average net impact of bandwagons on regime change. The decomposition is an algebraic identity and does not depend on distributional assumptions. In the Appendix, we further verify that the signs of $T_\mathcal{I}$, $T_\mathcal{T}$ and $T_\mathcal{B}$ indeed have the comparative statics we would expect from section 3.

While Lemma 5 allows us to restrict attention to persuasion, targeting and bandwagons, the following Corollary shows that they always conflict in the most studied version of the model, where f is linear:

Corollary 2. Suppose $f(A) = A$. Two of $T_\mathcal{I}(\rho)$, $T_\mathcal{T}(\rho)$ and $T_\mathcal{B}(\rho)$ have opposing signs, $\forall \rho$.

Indeed, the persuasion and targeting roles can only be mutually positive if $\theta^\star_\emptyset > \frac{1}{2}$ (Propositions 1 & 2), the same circumstances under which the bandwagon role is negative (Proposition 3).

However as the next Proposition shows, we can evaluate this trade-off for all increasing C^2 functions f in the limit as the noise of the public signal gets large. We think of this as the effect of a small amount of public information on regime change.

Proposition 4. $\lim_{\sigma_\eta\to\infty} T(\rho) \to 0^+$ if and only if

$$\underbrace{\left(2-\frac{\partial\theta^\star_\emptyset}{\partial\mu}\right)\left(-\frac{\partial\theta^\star_\emptyset}{\partial\mu}\right)\left(\overline{\theta}-\theta^\star_\emptyset\right)}_{\textit{Targeting Role}}\overbrace{-2\omega_\emptyset\left(1-\omega_\emptyset\right)\frac{\partial\theta^\star_\emptyset}{\partial\omega_\emptyset}}^{\textit{Persuasion Role}}-\underbrace{\sigma^2_\theta\left(v-1\right)\frac{\partial t}{\partial x^\star_\emptyset}\frac{\partial\theta^\star_\emptyset}{\partial\mu}}_{\textit{Bangwagon Role}}>0 \tag{11}$$

where $t\left(x^\star,\theta^\star\right):=1-\frac{\partial\theta^\star}{\partial\mu}=1+\frac{1-\omega_\emptyset}{\omega_\emptyset v-1}>0$, $\frac{\partial\theta^\star}{\partial\omega_\emptyset}=-\frac{x^\star_\emptyset-\overline{\theta}+\frac{\sigma_\epsilon}{2}\omega_\emptyset^{-\frac{1}{2}}\Phi^{-1}(c)}{\omega_\emptyset v-1}$ and $v=1+\frac{\sigma_\epsilon}{f'.\phi\left(\frac{x^\star-\theta^\star}{\sigma_\epsilon}\right)}$.

Proposition 4 provides conditions under which public information increases the probability of regime change, as $\sigma^2_\eta\to\infty$. Since public signals are taken to the fully imprecise limit, $T(\rho)$ converges to 0. The Proposition tells us whether this convergence takes place from above (so $T(\rho)>0$, for large enough σ^2_η) or below and distinguishes the relative contributions of the persuasion, targeting and bandwagon roles. The proof evaluates the limiting derivative of $T(\rho)$ with respect to σ^2_η, making use of the Dominated Convergence Theorem after an appropriate reformulation of the integrand.[25]

Equation (11) shows that persuasion, targeting and bandwagons all generally remain important in the limit. The persuasion role is proportional to

$$-2\omega_\emptyset\left(1-\omega_\emptyset\right)\frac{\partial\theta^\star_\emptyset}{\partial\omega_\emptyset}\phi\left(\frac{\theta^\star_\emptyset-\overline{\theta}}{\sigma_\theta}\right) \tag{12}$$

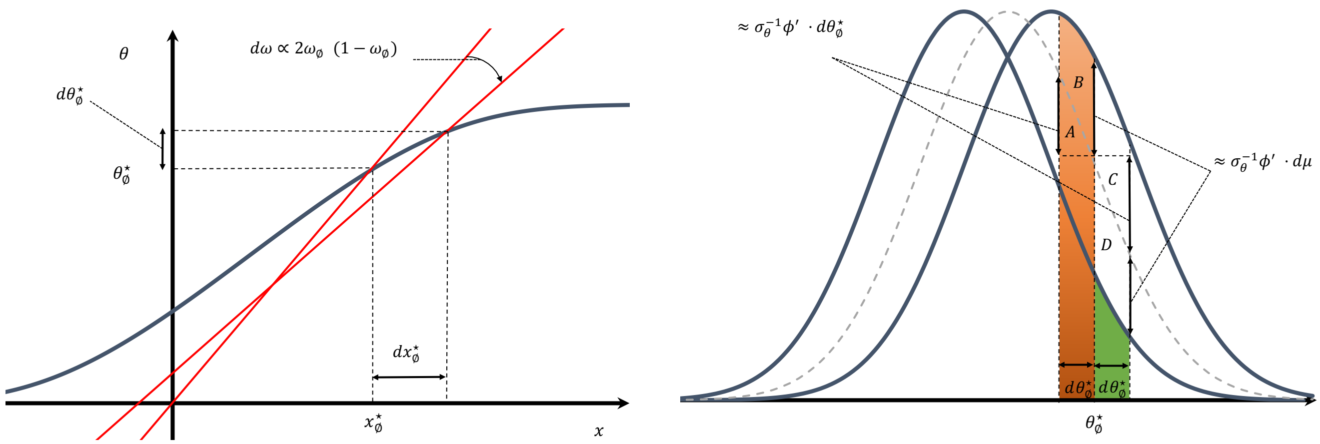
where $-\frac{\partial\theta^*_\emptyset}{\partial\omega}$ is the marginal effect of a reduction in ω on the equilibrium threshold under opacity, $2\omega_\emptyset\left(1-\omega_\emptyset\right)$ measures the marginal effect of the public signal on weight $\omega_\emptyset$ and $\phi\left(\frac{\theta^\star_\emptyset-\overline{\theta}}{\sigma_\theta}\right)$ is the density of I's resilience at $\theta^\star_\emptyset$. (12) is therefore the effect of information on regime change via systematic persuasion of the marginal agent, as we expect from section 3.1 (see Figure 2 (a)). In particular the sign of (12) depends solely on how the marginal agent's beliefs respond to information, via a reduction in the weight ω placed on his type.

The targeting role is proportional to

$$\left(2-\frac{\partial\theta^\star_\emptyset}{\partial\mu}\right)\left(-\frac{\partial\theta^\star_\emptyset}{\partial\mu}\right)\left(\overline{\theta}-\theta^\star_\emptyset\right)\phi\left(\frac{\theta^\star_\emptyset-\overline{\theta}}{\sigma_\theta}\right) \tag{13}$$

(13) is best understood by looking at Figure 2 (b). Consider two public signals $y_+=\overline{\theta}+\gamma$, $y_-=\overline{\theta}-\gamma$ which shift the public posterior μ_y by $d\mu$, $-d\mu$ and consequently shift the threshold $\theta^\star_\emptyset$ by $-d\theta$, $d\theta$ respectively. The excess impact of signal $\overline{\theta}+\gamma$ over $\overline{\theta}-\gamma$ on regime

[25] In particular, we make use of a positive scaling factor such that the rescaled derivative does not shrink 'too fast' in the limit.



(a) (b)

Figure 2: (a) The (limiting) persuasion role of public information depends on $\frac{\partial\theta^*_\emptyset}{\partial\omega_\emptyset}$; (b) The (limiting) targeting role of public information is given by Area $A+B+C+D$.

change is given by the sum of areas A, B, C and D:

$$\sigma_\theta^{-1}\phi'\left(\frac{\theta^\star_\emptyset-\overline{\theta}}{\sigma_\theta}\right)(-d\theta)^2+2\sigma_\theta^{-1}\phi'\left(\frac{\theta^\star_\emptyset-\overline{\theta}}{\sigma_\theta}\right)(-d\theta)\,d\mu$$

Noting that $\phi'(u)=-u\phi(u)$, (13) is the targeting effect for a change $d\mu=\sigma_\theta^2$, which is indeed proportional to the effect of public information on μ. Since $\frac{\partial\theta^\star_\emptyset}{\partial\mu}<0$, the sign of the targeting role is determined solely by the sign of $\left(\overline{\theta}-\theta^\star_\emptyset\right)$. If $\overline{\theta}>\theta^\star_\emptyset$ then signal $y=\overline{\theta}+\gamma$ incentivizes the marginal agent to attack when the fundamental impact of larger attacks on the regime's survival is actually low, and discourages him when the marginal impact of larger attacks is actually high.

Finally, the magnitude of the bandwagon role is given by $-\sigma_\theta^2(v-1)\frac{\partial t}{\partial x}\frac{\partial\theta^\star_\emptyset}{\partial\mu}$. Direct calculation shows that $v=\frac{1}{\theta'_P(x)}>1$, the multiplicative inverse of the slope of $\theta_P(x)$. Since $t(x^\star,\theta^\star)$ only depends on $\theta^\star_\emptyset$ through v, the derivative $\frac{\partial t}{\partial x}$ is tightly connected to the curvature of $\theta_P(x)$. Indeed, $\frac{\partial t}{\partial x^\star}$ always takes the sign of $\theta''_P\left(x^\star_\emptyset\right)$. Since $\frac{\partial\theta^\star_\emptyset}{\partial\mu}<0$, the bandwagon effect contributes positively to regime change only if θ_P is locally convex, as we expect from section 3.3.

The following Corollary is immediate from the above discussion:

Corollary 3. Under assumption (2):

1. $T_{\mathcal{T}}(\rho)>(<)0$ if $\overline{\theta}>(<)\theta^\star_\emptyset$;

2. $T_{\mathcal{I}}(\rho) > (<) 0$ if $\overline{\theta} < (>) \theta^{\star}_{\emptyset} - \frac{\sigma_{\epsilon}}{2}\omega_{\emptyset}^{\frac{1}{2}}\Phi^{-1}(c)$

3. $T_{\mathcal{B}}(\rho) > (<) 0$ if $\theta''_P(x^{\star}_{\theta}) > (<) 0$

The first and third implications are immediate from Proposition 4, while the second follows after making use of (5) to slightly rewrite $-\frac{\partial \theta^{\star}_{\emptyset}}{\partial \omega_{\emptyset}}$. As we expect from sections 3.1 and 3.2, the targeting role is positive when $\overline{\theta}$ is sufficiently high relative to $\theta^{\star}_{\emptyset}$, while the persuasion role is positive when $\overline{\theta}$ is suffciently low. In other words, there is a systematic tendency for these roles to conflict, with the severity of conflict measured by $\left|\omega_{\emptyset}^{\frac{1}{2}}\Phi^{-1}(c)\right|$.

Importantly, both the targeting and persuasion roles are purely informational effects, based on how the marginal agent's beliefs relate to the fundamentals of the economy. Moreover, the curvature of the 'production function', $\theta''_P(x)$ does not explicitly appear in either (12) or (13).[26] By contrast, the sign of the bandwagon effect solely relies on asymmetries in equilibrium outcomes following signals $y = \overline{\theta} \pm \gamma$, as captured in the curvature of $\theta_P(x)$.

We now evaluate the impact of public information on regime change in two different settings: First, we study a class of piecewise linear models, in which only the pure informational effects are present a.e. As we show in the online appendix, the former class of models naturally arises in situations where I's resilience depends on his ability to recruit agents to its defence.[27] We then consider settings where bandwagons are generically present.

Comparative statics: $T(\rho)$ with small bandwagons

We now suppose that the solution curve to (4) takes the following form:

$$\theta_P(x) = \begin{cases} mx_L + b & , \ x \leq x_L \\ mx + b & , \ x_L \leq x \leq x_H \\ mx_H + b & , \ x_H \geq x \end{cases}$$

where $x_L < x_H \in \mathbb{R}$ and $0 < m < \omega_{\emptyset}$, the equivalent of the uniqueness condition (2). Here $\theta_P(x)$ is piecewise linear and continuous, with constant-valued left and right pieces.[28] m can

[26]The first derivative, θ'_P, however is important for the magnitudes of both effects since $\frac{\partial \theta^{\star}}{\partial \mu}$, $\frac{\partial \theta^{\star}}{\partial \omega}$ both depend on v.

[27]For example, in problems of collective bargaining where a large number of union employees face an employer (a firm or government body) who may pay for alternative short-term labour in the event of a strike.

[28]This rules out the possibility of two extreme equilibria, in which *(i)* all types attack I is overthrown with certainty, and *(ii)* no types attack and I survives with certainty. However, this is not necessary for our results. In particular, we recover the same comparative statics predictions uniquely even if $\theta_P(x) \to \pm\infty$ as $x \to \pm\infty$, so long as each linear section of θ_P sastisfies $\theta'_P < \omega_{\emptyset}$ and we select only equilibria that are stable under best response dynamics.

be interpreted as a measure of the marginal returns to larger attacks. It is the rate at which the threshold $\theta^\star$ responds to a more aggressive cut-off strategy on the increasing section of θ_P. Since we allow m, b to be arbitary, the results of this section are easily extendable to continuous piecewise linear functions with arbitarily many pieces.

When $\theta_P(x)$ is piecewise linear, we can solve directly for equilibrium under opacity by substituting $\theta^\star_\emptyset = \theta_P\left(x^\star_\emptyset\right)$ into (5) at $z = \emptyset$. In particular, focusing first on the most interesting case where $x_L < x^\star_\emptyset < x_H$, $\theta^\star_\emptyset$ satisfies

$$\left(\frac{\omega_\emptyset - m}{m}\right)\theta^\star_\emptyset = \frac{\omega_\emptyset b}{m} - (1-\omega_\emptyset)\,\overline{\theta} - k \tag{14}$$

where $k = \sigma_\epsilon \omega_\emptyset^{\frac{1}{2}} \Phi^{-1}(c)$ is an increasing function of c. While θ_P is not everywhere differentiable, a simiplified form of equation (4) still determines the limiting sign of $T(\rho)$ whenever $x^\star_\emptyset \neq x_L, x_H$.[29] For $x_L < x^\star_\emptyset < x_H$, we have $\frac{\partial t}{\partial x^\star_\emptyset} = 0$ (since $\theta''_P\left(x^\star_\emptyset\right) = 0$) and $v \equiv \theta'_P(x) = \frac{1}{m}$. Thus (11) becomes

$$0 \;<\; \left(2 + \frac{1-\omega_\emptyset}{\omega_\emptyset v - 1}\right)\left(\frac{1-\omega_\emptyset}{\omega_\emptyset v - 1}\right)\left(\overline{\theta} - \theta^\star_\emptyset\right) + 2\omega_\emptyset\,(1-\omega_\emptyset)\,\frac{x^\star_\emptyset - \overline{\theta} + \frac{\sigma_\epsilon}{2}\omega_\emptyset^{-\frac{1}{2}}\Phi^{-1}(c)}{\omega_\emptyset v - 1} \tag{15}$$

Noticing from (5) that $\theta^\star_\emptyset - \overline{\theta} = \omega_\emptyset\left(x^\star_\emptyset - \overline{\theta}\right) + \sigma_\epsilon \omega_\emptyset^{\frac{1}{2}} \Phi^{-1}(c)$ and substituting (14) into (15), $T(\rho) > 0$ for all σ^2_η large if

$$k\left(1 - \omega_\emptyset - \left(\frac{m - \omega_\emptyset}{m}\right)^2\right) \geq (1-\omega_\emptyset)\left(\frac{\omega_\emptyset b}{m} - \omega_\emptyset\left(\frac{1-m}{m}\right)\overline{\theta}\right) \tag{16}$$

Based on (16), the comparative statics of transparency are:

Corollary 4. For σ^2_η sufficiently large, $T(\rho)$ has at most a single crossing with 0 in parameters $\overline{\theta}$, c on $x^\star_\emptyset \in (x_L, x_H)$. $T(\rho)$ crosses from below in $\overline{\theta}$ and crosses from below (above) in c if

$$m > (<) 1 - (1-\omega_\emptyset)^{\frac{1}{2}}$$

Corollary 4 presents comparative statics of $T(\rho)$ in $\overline{\theta}$, c on the increasing portion of $\theta_P(x)$. Although condition (16) is linear in $\overline{\theta}$, k, the constraint $x^\star_\emptyset \in (x_L, x_H)$ means that $T(\rho)$ may not have any crossing on the increasing portion of θ_P as we vary $\overline{\theta}$ or c. Suppose that there is an interior crossing in $\overline{\theta}$ for some fixed c. Then the corollary states that $T(\rho)$ is positive (negative) when $\overline{\theta}$ is sufficiently high (low).[30] This happens because targeting is

[29] Proof available on request.

[30] When $x^\star_\emptyset = x_L$, the upward 'kink' in $\theta_P(x)$ can be shown to imply that bandwagon effects dominate,

always more responsive to $\overline{\theta}$ than persuasion.

Comparative statics in c are more subtle. In particular, whether the crossing happens from above or below depends on the slope of θ_P.[31] When m is small, $T(\rho)$ is positive for sufficiently high c. But when m is low enough, $T(\rho)$ is positive only when c is sufficiently low. The cost parameter c affects ability to persuade the marginal agent directly, but also indirectly influences the magnitude of the targeting role through $\left(\theta^\star_\emptyset - \overline{\theta}\right)$. When m is small, the equilibrium $\theta^\star_\emptyset$ is relatively insensitive to changes in c and therefore persuasion becomes more important for comparative statics, and vice versa.

Interestingly, when $m > 1 - (1-\omega_\emptyset)^{\frac{1}{2}}$, the overall impact of transparency tends to be positive when $\overline{\theta}$ and/or c are high, the circumstances where the persuasion role is typically negative. To see this, suppose $c = \frac{1}{2}$ and consider comparative statics in $\overline{\theta}$. From (15), $T(\rho) > 0$ if $\overline{\theta} > \theta^\star_\emptyset$. But recalling Corollary 3, these are the same conditions under which $T_{\mathcal{I}}(\rho) < 0$. Similarly, $T(\rho) < 0$ while $T_{\mathcal{I}}(\rho) > 0$ for $\theta^\star_\emptyset < \overline{\theta}$. Moreover because targeting is more sensitive to $\overline{\theta}$ than persuasion, this behaviour is most likely for extreme values of $\overline{\theta}$. By contrast, the equivalent result in c requires m sufficiently large.

Corollary 4 highlights the difference between our model of public information and models which study the effect of prior informativeness on regime change. For instance in Iachan and Nenov [2015], a lower variance prior increases regime change if and only if the chances of success are already sufficiently high.[32] By contrast we find the opposite comparative static, so long as m is sufficiently high. The difference occurs because an increase in prior precision has a persuasion but no targeting effect.

Comparative statics: $T(\rho)$ with large bandwagons

In general, bandwagon effects can play an important role in outcomes. In particular, we now return to the more general setting of Section 2.1 and consider comparative statics results in $\overline{\theta}$. In this setting, it is possible for $T(\rho)$ to have multiple crossings in $\overline{\theta}$, even for $f(A) = A$, as the interplay between the persuasion, targeting and bandwagon roles can be complex. Nonetheless as the next result shows, bandwagons unambiguously dominate as $\overline{\theta} \to \pm\infty$.

Proposition 5. Suppose $f : [0,1] \to \mathbb{R}_+$ is a C^2 function. For any c, there exist thresholds

unambiguously causing $T(\rho) > 0$ in the limit. Similarly, $T(\rho) < 0$ for $x^\star_\emptyset = x_H$. Nonetheless, $T_{\mathcal{B}}(\rho) = 0$ generically for θ_P piecewise linear. For $x^\star_\emptyset < x_L, > x_H$, $T(\rho)$ is always 0 in the limit, which can be seen by evaluating (16) at $m = 0$.

[31] Note that the set $\left\{m : \omega_\emptyset > m > 1 - (1-\omega_\emptyset)^{\frac{1}{2}}\right\}$ is nonempty for all $0 < \omega_\emptyset \leq 1$, since $g(\omega) := 1 - (1-\omega)^{\frac{1}{2}}$ is convex with $g(0) = 0$, $g(1) = 1$.

[32] Other global games of regime change which consider comparative statics in prior informativeness include Hellwig [2002], Metz [2002], Morris and Shin [2004], Bannier and Heinemann [2005], Szkup and Trevino [2015]

$\theta_L \leq \theta_H$ such that $sign\left(T\left(\rho\right)\right) = sign\left(T_{\mathcal{B}}\left(\rho\right)\right)$ for $\overline{\theta} \leq \theta_L$, $\overline{\theta} \geq \theta_H$ where

1. $T_{\mathcal{B}}\left(\rho\right) > 0$ for $\overline{\theta} \geq \theta_H$

2. $T_{\mathcal{B}}\left(\rho\right) < 0$ for $\overline{\theta} \leq \theta_L$.

Proposition 5 shows that bandwagons are typically important when $\overline{\theta}$ is sufficiently extreme. For instance when $\overline{\theta}$ is high enough, $\theta_P\left(x\right)$ becomes convex and bandwagons become positive. In this case, aggregate attacks under opacity are so small that signals of I's strength have limited further downside while signals of weakness have substantial upside. In the Appendix, we show that while $T_{\mathcal{I}}$, $T_{\mathcal{T}}$ and $T_{\mathcal{B}}$ all approach 0 as $\overline{\theta}$ becomes extreme, $T_{\mathcal{B}}$ diminishes relatively slowly so long as f'' does not diverge too quickly as $A \to 0, 1$. A simple sufficient condition for this is that f is C^2 on the compact set $[0, 1]$ and therefore bounded.

In the next section, we turn to the decomposition of Lemma 5 to understand the impact of transparency on regime change in several applications which emphasize different models of publicly available information.

5 Transparency and the Probability of Regime Change: Applications and Extensions

We now study the overall impact of transparency $T\left(\overline{\theta}, c, f\right)$ in several applications, in which we consider alternative structures of public information. In the first extension, we study public signals which only communicate about the fundamentals of the economy without revealing anything on the opinions of others. We then consider what happens when public signals cannot be distinguished from private information. Lastly, we look at coarse binary signals which effectively only allow agents to rule out extreme events. As we discuss in each section, these models of public information take on varying degrees of prominence depending on the economic environment. For instance, binary public signals naturally arise in financial stress tests where only a pass-fail result is communicated.

Although these public information structures are superficially very different, we show that the tools developed in section 3 can be used to understand the impact of transparency in all three applications. In particular, in each case we show that only one of the roles of public information identified above dominates and derive comparative statics results which can help inform policy on transparency in macroeconomic, financial and political economy applications. Studying the impact of transparency in different settings allows us to identify the driving forces behind exisiting results in the literature.

5.1 Uncorrelated Public-Private Signals

In section 3, we modelled both public and private information as noisy signals of the same underlying state variable, θ. In this case y was correlated with both θ and x, meaning that transparency provided information to agents on two strategically important variables: *(i)* I's fundamental strength, and *(ii)* the likely beliefs of other agents.[33] In particular we argued that information about the $\emptyset$-marginal agent's willingness to attack was a key driver of the persuasion role of transparency.

We now consider the implications of releasing public information that is truly independent of private signals. In many settings of interest the information available for public dissemination is indeed largely unrelated to the private beliefs of the agents. For example, central banks have information on their capacity to withstand speculative currency attacks. But a commitment to publicizing reserves is not likely to provide any spillover information on investors' views about macroeconomic risks or the impact of monetary contraction on that economy, which might affect the willingness of large speculators to attack or the bank to defend a rate peg, respectively.

Specifically, we extend the model of section 2 by allowing the state to be the sum of two components:[34]

$$\theta = \tilde{\theta} + \hat{\theta}$$

where $\tilde{\theta}$, $\hat{\theta}$ are normally and independently distributed random variables, with $\tilde{\theta} \sim N\left(\overline{\theta}, \tilde{\sigma}^2\right)$ and $\hat{\theta} \sim N\left(0, \hat{\sigma}^2\right)$. Private and public signals, written $\tilde{x}_i$ and $\hat{y}$ respectively, are given by

$$\tilde{x}_i = \tilde{\theta} + \epsilon_i$$

$$\hat{y} = \bar{\theta} + \hat{\theta} + \eta$$

where ϵ_i, η are as defined in section 2. Focusing on cut-off strategies, the equilibrium conditions become

$$\theta_z^\star = (f \circ \Phi)\left(\frac{\tilde{x}_z^\star - k_z\theta_z^\star - (1 - k_z(1-\rho_z))\,\overline{\theta} + k_z\rho_z\hat{y}}{\left(\sigma_\epsilon^2 + (1-k_z)\,\tilde{\sigma}^2\right)^{\frac{1}{2}}}\right) \tag{17}$$

$$c = \Phi\left(\frac{\theta_z^\star - \nu\tilde{x}_z^\star - \rho_z\hat{y} - (1-\nu-\rho_z)\,\overline{\theta}}{\left(\nu\sigma_\epsilon^2 + (1-\rho_z)\,\hat{\sigma}^2\right)^{\frac{1}{2}}}\right) \tag{18}$$

[33] Indeed, $cov\left(\theta, y \mid x_i\right) = var\left(\theta \mid x_i\right) > 0$ and $cov\left(x_j, y \mid x_i\right) = cov\left(\theta + \epsilon_j, y \mid x_i\right) = var\left(\theta \mid x_i\right) > 0$ for all $j \neq i$.

[34] Bouvard et al. [2015] consider a similar information structure in a Diamond-Dybvig bank run game with binary public signals.

where $\nu = \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2+\sigma_\epsilon^2}$, $k_\emptyset = \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2+\hat{\sigma}^2}$, $\rho_\emptyset = 0$ and $k_z = \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2+(1-\rho_z)\hat{\sigma}^2}$, $\rho_z = \frac{\hat{\sigma}^2}{\hat{\sigma}^2+\sigma_\eta^2}$ for $z \in \mathbb{R}$. Equations (17) - (18) are the approriate analogues of (4) - (5) when $\tilde{x}$, $\hat{y}$ are uncorrelated as above. Given this, we can establish the following:

Proposition 6. Suppose public and private information are uncorrelated and furthermore, $\tilde{\sigma}^2 \to 0$, $\lim \frac{\sigma_\epsilon^2}{\tilde{\sigma}^2} \leq p$, for some $p \in \mathbb{R}_+$. Then

$$\lim_{\tilde{\sigma}^2 \to 0} sign\left(T\left(\rho\right)\right) = \lim_{\tilde{\sigma}^2 \to 0} sign\left(T_{\mathcal{B}}\left(\rho\right)\right)$$

Moreover, $T\left(\overline{\theta}, c, f\right) \geq 0$ iff $\overline{\theta} \geq x_\emptyset^\star$.

Proposition 6 considers the limiting case as *(i)* the variation in the private component of θ is small and *(ii)* the quality of private information about this component is high. In this case, the Proposition tells us that the overall impact of transparency is determined by the bandwagon effect. Intuitively when the precision of private information is high, disagreements among agents are rare since all agents are highly likely to draw $\tilde{x}$ values near to $\tilde{\theta}$. Moreover when $\tilde{\sigma}^2$ is small $\tilde{\theta}$ is likely to be close to $\bar{\theta}$. Thus a cut-off strategy to attack iff $x \leq \tilde{x}_z$ either leads to insignificantly small attacks ($\tilde{x}_z^* < \overline{\theta}$) or essentially the entire population attacking ($\tilde{x}_z^* > \overline{\theta}$). In the former case, $\theta_z^* \to 0$, while in the latter, $\theta_z^* \to 1$. This implies that bandwagon effects are dominant - public information only has a marginal impact on regime change when it causes x_y^* to cross the $\overline{\theta}$ threshold for some values of y.

Given the above discussion, the overall impact of transparency is clearly positive when $x_\emptyset^* < \overline{\theta}$ since under opacity, nobody attacks in the limit. By providing the agents with a signal $\hat{y}$ which shifts their beliefs in the strength of the regime, transparency can then incentivize the group to attack in large numbers for at least some public realizations.

Finally, we can characterize the conditions under which $\tilde{x}_\emptyset^* \leq \overline{\theta}$. Since the solution curve to (17) becomes a step function in the limit, it is in general not possible to rule out multiple equilibria for some parameter values. Nonetheless, as we show in the Appendix, there can be at most 2 stable cut-off equilibria.[35] Specifically, we have

Lemma 6. Consider an opaque continuation game. In the limit as $\tilde{\sigma}^2 \to 0$, $\lim \frac{\sigma_\epsilon^2}{\tilde{\sigma}^2} \leq p$, for some $p \in \mathbb{R}_+$:

1. There is a unique opaque equilibrium with $\tilde{x}_\emptyset^* < \overline{\theta}$ iff $\overline{\theta} + \left(\nu\sigma_\epsilon^2 + \hat{\sigma}^2\right)^{\frac{1}{2}} \Phi^{-1}\left(c\right) > 1$

2. There is a unique opaque equilibrium with $\tilde{x}_\emptyset^* > \overline{\theta}$ iff $\overline{\theta} + \left(\nu\sigma_\epsilon^2 + \hat{\sigma}^2\right)^{\frac{1}{2}} \Phi^{-1}\left(c\right) < 0$

3. There are three equilibria, $\tilde{x}_{\emptyset,1}^* < \overline{\theta}$, $\tilde{x}_{\emptyset,2}^* = \overline{\theta}$, $\tilde{x}_{\emptyset,3}^* > \overline{\theta}$ iff $0 \leq \overline{\theta} + \left(\nu\sigma_\epsilon^2 + \hat{\sigma}^2\right)^{\frac{1}{2}} \Phi^{-1}\left(c\right) \leq 1$

[35]We show that the potential for multiplicity does not alter the conclusions of Proposition 6 so long as the continuation equilibrium under opacity is in cut-off strategies.

5.2 Transparency when Public Component is Unobserved

In many economic environments, public signals are open to interpretation and can exacerbate differences of opinion. For instance, macroeconomic data releases such as unemployment figures can result in larger disagreements among analysts about the state of the economy. In the model of section 2, public signals are however commonly observed. In particular, even though agents have heterogeneous private beliefs they agree on the realization of the public signal. As we showed in section 3.1, in that case public signals cause agents' beliefs to conform.

To allow transparency to exacerbate differences of opinion, we adjust the model of section 2 as follows: Under transparency, we let the common signal be perfectly precise, $\tilde{y} = \theta$, but only allow agents to observe a single private signal, $\tilde{x}(i)$, given by

$$\tilde{x}(i) = \kappa x(i) + (1-\kappa)\tilde{y} = \theta + \kappa\epsilon(i)$$

where $x(i)$ is as defined in section 2 and $\kappa \in (0,1)$.[36] Under opacity the model continues as before. Notice that $\tilde{x}(i)$ is more precise than $x(i)$ (since $\kappa < 1$) so that all agents are commonly understood to be better informed under transparency than under opacity. However since agents only observe $\tilde{x}(i)$ under transparency, they cannot perfectly infer how much of their information is common, due to $\tilde{y}$, and how much is idiosyncratic, $x(i)$. Thus, equilibrium strategies under transparency can no longer be conditioned on the particular value of any public signal. As we show, this diminishes the roles that targeting and bandwagon effects can play.

In this case, the equilibrium conditions become

$$\theta_z^\star = f\left(\Phi\left(\frac{x_z^\star - \theta_z^\star}{\kappa_z\sigma_\epsilon}\right)\right) \quad (19)$$

$$c = \Phi\left(\frac{\theta_z^\star - \tilde{\omega}_z x_z^\star - (1-\tilde{\omega}_z)\overline{\theta}}{\sigma_\epsilon\sqrt{\tilde{\omega}_z}}\right) \quad (20)$$

where $\kappa_\emptyset = 1$, $\kappa_y = \kappa < 1$ and $\tilde{\omega}_z = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \kappa_z^2\sigma_\epsilon^2}$, and we can establish the following result:

Proposition 7. Suppose agents cannot distinguish common and idiosyncratic beliefs. Then, $T_{\mathcal{T}}(\rho), T_B(\rho) = 0$ and

$$T(\rho) > 0$$

if and only if $A_y(\theta_\emptyset^\star) > A_\emptyset(\theta_\emptyset^\star)$, which occurs when $\overline{\theta}$ and/or c are sufficiently large.

[36] The conditional variance of beliefs across types, $var(\mathbb{E}[\theta \mid x] \mid \theta)$, is higher under transparency if $\kappa > \frac{\sigma_\theta}{\sigma_\epsilon}$. A necessary condition for thsi to be the case is therefore $\sigma_\epsilon > \sigma_\theta$.

Proposition 7 tells us that when agents cannot distinguish commonly observed signals from their own private signals, public information cannot induce bandwagon or targeting effects. Since agents cannot use $\tilde{y}$ to anticipate when others are likely to attack, public information cannot be used in these ways. Instead, the effect of 'public' information on regime change is driven entirely by its effect on attack size at the margin. The incentives of type $x^\star_\emptyset$ are an important driver of these larger attacks. But in this case better information also reduces the dispersion of agent's types, making type $x^\star_\emptyset$ more of an outlier in the population under public information. However, we can still interpret Proposition 7 as showing conditions under which the persuasion role dominates. Thinking of the $\emptyset$-marginal agent as the type whose rank in the distribution $\tilde{x}$ of beliefs under transparency is equal to that of $x^\star_\emptyset$ under opacity, the Proposition states that regime change increases if and only if this 'marginal' agent becomes strictly incentivized to attack under public information. [37]

Despite the dominance of the persuasion role of transparency, the comparative statics of Proposition 7 appear to contrast with the findings of Proposition 1. The reason for this difference is simple: When agents can distinguish public from private signals, transparency induces them to put less weight on their own types. Conversely, when agents cannot distinguish the common from idiosyncratic content of signals, transparency causes them to put more weight on their own types. Thus, a relatively pessimistic $\emptyset$-marginal agent becomes less so under transparency in the former case, but more so in the latter. Recalling the definition of ω_z, it is easy to see that $\omega_y < \omega_\emptyset$ while $\tilde{\omega}_y > \tilde{\omega}_\emptyset$.

Along with Corollary 4, this example helps illustrate the distinction between the effects of public information and the precision of the prior on the chances of regime change.[38] It is well known that a more precise prior increases the chances of regime change when $\overline{\theta}$ is low. However, the comparative statics of Corollary 4 and Propositions 5 and 7 show that public information increases the chances of regime change when $\overline{\theta}$ is low, regardless of whether agents can disentangle the public and private components of their information.

5.3 Binary Public Signal

We now depart from the assumption of normality and treat public information as a coarse, binary signal with the following threshold form. In this context, a public signal y is a random variable taking values in $\{0, 1\}$, where $y = 1$ iff $\theta > \bar{\theta}$. A signal $y = 1$ (a 'Pass') makes it common knowledge that I is stronger than the mean. Likewise, a signal $y = 0$ ('Fail')

[37] By contrast, in sections 2-4, this alternative interpretation of the 'marginal' agent is equivalent to $x^\star_\emptyset$.

[38] Iachan and Nenov [2015], Szkup and Trevino [2015], Bannier and Heinemann [2005], Metz [2002] consider the role of private signal precision on regime change, and in some instances reference the role of changes in the relative precision of the prior.

makes it common knowledge that I is weaker than the mean.[39] We interpret this kind of binary signal as a financial stress test, revealing a coarse ranking of the bank's resilience to potential runs. We also assume for clarity of exposition that $f(0) < \bar{\theta} < f(1)$.

The threshold public information structure breaks two-sided limit dominance under transparency. As is standard in models of one-sided limit dominance, complete unravelling occurs.[40] To see this, consider the case where $y = 1$ is observed. Notice that it is common knowledge among the agents that $\theta \geq \bar{\theta} > f(0)$. But if $\theta > f(0)$, no agent can ever have a dominant strategy to attack - he would need to expect that at least $f^{-1}\left(\bar{\theta}\right)$ agents will attack the regime before he will even consider doing so. By contrast, the event $\theta \in (f(1), \infty)$ that I can withstand any run is still possible when $y = 1$ and in particular those agents with high enough x will have a dominant strategy to refrain. Since some agents have a dominant strategy to refrain and no-one has a dominant strategy to attack, there is always an equilibrium in which $x_1^* = -\infty$ and $\theta_1^* = f(0)$. Therefore under transparency I is overthrown iff $\theta \leq \bar{\theta}$ (that is, on observing 'Fail' news) while under opacity the incumbent is overthrown iff $\theta \leq \theta_\emptyset^*$.[41] Formally:

Proposition 8. Suppose public information is binary. Then

$$sign\left(T\left(\overline{\theta}, c, f\right)\right) = sign\left(T_E\left(\overline{\theta}, c, f\right)\right)$$

Moreover, $T\left(\overline{\theta}, c, f\right) \geq 0$ iff $\overline{\theta} \geq \theta_\emptyset^\star$.

When public information is binary, the targeting role of transparency is dominant. When $\theta_\emptyset^* < \bar{\theta}$, I is most vulnerable to a change in attack size if $\theta \in \left[\theta_\emptyset^*, \overline{\theta}\right]$. In particular, types $\theta \geq \overline{\theta}$ survive even under opacity and continue to do so under transparency (since $y = 1$ for these types). Similarly, types below $\theta_\emptyset^*$ would be overthrown both under opacity and transparency. But while types with $\theta \in \left[\theta_\emptyset^*, \overline{\theta}\right]$ survive under opacity, they are overthrown under transparency because signal $y = 0$ leaves them vulnerable to a larger attack. It is easy to see in this case that transparency increases the ex ante probability of regime change iff $\theta_\emptyset^* < \bar{\theta}$.

When public signals take a threshold form, (10) becomes

$$\frac{1}{2}\left[\Pr\left(\theta_0^\star, \theta_\emptyset^\star; \mu_{\theta|y=1}\right) - \Pr\left(\theta_\emptyset^\star, \theta_1^\star; \mu_{\theta|y=0}\right)\right] \tag{21}$$

[39] Angeletos et al. [2007] develop a dynamic model in which a binary public signal arises endogenously. However, their aim is not to evaluate the ex ante effect of this signal on the probability of regime change.

[40] See Baliga and Sjostrom [2004], for example.

[41] The proof for Proposition 8 extends the argument to any exogenously given threshold public information $\hat{\theta}$ and to cases where threshold information might induce multiple equilibria. Interestingly, the result is not vulnerable to multiplicity introduced in the latter case.

which simplifies to $\frac{1}{2} - \Phi\left(\frac{\theta^\star_\emptyset - \bar{\theta}}{\sigma_\theta}\right)$. Just as in the definition of efficacy in (6), (21) compares the average net impact on the probability of regime change across symmetric realizations of good and bad news. In particular, notice that we set up our public signal to be ex ante symmetric about $\overline{\theta}$. In this case, there is no persuasion role for transparency. Moreover, the role of bandwagons turns out to be subsidiary to the targeting effect.

6 Conclusions

In this paper we study the *ex ante* impact of an informative public signal on the probability of regime change in a global game, accounting for uncertainty over what will be communicated. We show that there is a fundamental tension between two key roles of public information: access to public signals can bias groups towards attacking in larger numbers, but often at the cost of targeting those attacks wastefully. In particular while transparency improves the targeting of attacks when the probability of regime change is otherwise low, it only systematically incentivizes larger aggregate attacks when the chances of regime change are already sufficiently high.

In the limit as public signals become imprecise, we are able to compare the relative contributions of persuasion and targeting to regime change. Their interaction generates new comparative statics results on the value of public information in coordination problems. For instance, when the incumbent is ex ante believed to be weak (strong), freedom of information always reduces (increases) the probability of regime change. This can happen despite simultaneously persuading agents into larger (smaller) attacks. By contrast, private costs of attacking ambiguously affect the value of transparency - lower costs imply transparency negatively affects regime change only if the marginal productivity of attacks is sufficiently high. Finally, we consider several alternative notions of transparency, in each case identifying the effect of public information on regime change.

Having identified that transparency influences regime change via these distinct channels, a natural question is: what are the welfare consequences? Regime changes brought about by large aggregate attacks may involve significantly different social costs from those brought about by better targeted attacks. A full exploration of the normative distinction between these roles in a model with a more detailed description of their social costs is an important avenue for future research.

Appendix

In this Appendix, we collect the proofs of the main results in the paper. Since equations (4) - (5) feature heavily below, we define $\theta_P(x)$ and $\theta_z(x)$, for any $z \in Z$, as the curves which solve these equations respectively. Written this way, a continuation equilibrium is a z-contingent pair (x_z^*, θ_z^*) such that $\theta_P(x_z^*) = \theta_z(x_z^*) = \theta_z^*$.

Proof of Lemma 1

Proof. We first establish existence & uniqueness of (x_z^*, θ_z^*) for any $z \in Z$, given c, $\overline{\theta}$ and noise parameters. By equation (4), $\theta_P(x) \geq f(0)$. Since f maps into $\mathbb{R}$, for any $A \in (0,1)$ we can calculate a lower bound $f(0) \geq f(A) - AF > -\infty$ where F is bounded above, given (2). A similar argument establishes that θ_P is bounded from above. Conversely $\theta_z(x)$ is linear in x since equation (5) can be written

$$\theta_z(x) = \omega_z x + (1-\omega_z)\mu_z + \sigma_\epsilon \sqrt{\omega_z}\Phi^{-1}(c) \tag{22}$$

Moreover, (2) implies that $\omega_z > 0$ and therefore from (22), $\lim_{x\to\pm\infty} \theta_z(x) = \pm\infty$. Combining these facts, in the limit we must have $\lim_{x\to\pm\infty}(\theta_P(x) - \theta_z(x)) = \pm\infty$.[42] But since both f and Φ are continuous and increasing functions, curves $\theta_P(x)$ and $\theta_z(x)$ are continuous and increasing in x. Thus a solution to system (4) - (5) must exist. We show uniqueness. Applying IFT to $\theta_P(x)$ yields

$$\frac{\partial\theta_P}{\partial x} = \frac{f'\left(\Phi\left(\frac{x-\theta_P}{\sigma_\epsilon}\right)\right)\cdot\phi\left(\frac{x-\theta_P}{\sigma_\epsilon}\right)}{f'\left(\Phi\left(\frac{x-\theta_P}{\sigma_\epsilon}\right)\right)\cdot\phi\left(\frac{x-\theta_P}{\sigma_\epsilon}\right)+\sigma_\epsilon} \tag{23}$$

while from (22), $\frac{\partial\theta_z}{\partial x} = \omega_z$. It is easy to verify that assumption (2) implies that $\frac{\partial\theta_P}{\partial x} \leq \frac{\partial\theta_z}{\partial x}$, for all $x \in \mathbb{R}$, $z \in Z$, with equality at most once. In other words, the function $\theta_P(x) - \theta_z(x)$ is strictly decreasing, establishing uniqueness of (x_z^*, θ_z^*). Given this, (x_z^*, θ_z^*) is continuously decreasing in c, $\overline{\theta}$ since *(i)* $\theta_z(x)$ decreases continuously in c, $\overline{\theta}$, $\forall x$; (ii) $\theta_P(x)$ does not vary with c, $\overline{\theta}$.[43] Finally, uniqueness of equilibrium then follows from standard iterated dominance arguments and is therefore omitted.[44]

We now establish Lemma 1.2. Notice first that when $x = \frac{1}{2}$, $\theta_P = f\left(\frac{1}{2}\right)$ uniquely solves (4). Now suppose $x < \frac{1}{2}$. Then since $\theta_P(x)$ is increasing, we must have $\theta_P(x) < f\left(\frac{1}{2}\right)$. By

[42] Since θ_P and θ_z are increasing in x, these limits must exist.

[43] Similarly, equilibrium decreases in μ_z for $z \in \mathbb{R}$.

[44] See for example, Angeletos et al. [2007].

(4), it must be that

$$\Phi\left(\frac{x-\theta_P(x)}{\sigma_\epsilon}\right)<\frac{1}{2}$$

from which $\theta_P(x)>x$, follows immediately given Φ is the standard normal c.d.f. The case $x>\frac{1}{2}$ is proved analogously.

To establish Lemma 1.3, fix some $\overline{\theta}\in\mathbb{R}$ and a point (x',θ') on curve $\theta_P(x)$. We can sustain (x',θ') as a unique continuation equilibrium following $z=\emptyset$ if we can find a c such that (22) holds at (x',θ'). From inspection of (22), this is clearly possibly since Φ^{-1} is a continuously increasing function with $\lim_{c\to 0}\Phi^{-1}(c)=-\infty$, $\lim_{c\to 1}\Phi^{-1}(c)=\infty$. □

Proof of Lemma 2

Proof. Recalling $y\sim N\left(\overline{\theta},\sigma_\theta^2+\sigma_\eta^2\right)$, it is clear that $y_n>\bar{\theta}\iff \Pr(y\le y_n)>\Pr(y>y_n)$. We now argue that this implies $\Pr_{\theta,y}\left[A_y(\theta)\ge A_\emptyset(\theta)\right]>\frac{1}{2}$. We can write the aggregate attack size following some signal $y'<y_n$, as a function of θ, $A_{y'}(\theta)=\Phi\left(\frac{x_{y'}-\theta}{\sigma_\epsilon}\right)>\Phi\left(\frac{x_{y_n}-\theta}{\sigma_\epsilon}\right)=A_\emptyset(\theta)$, where the inequality follows since $x_y^\star$ is decreasing in y (Lemma 1), and the final equality from $x_{y_n}^\star=x_\emptyset^\star$. A similar argument establishes$A_{y''}(\theta)\le A_\emptyset(\theta)$ for $y''\ge y_n$. Thus, $A_y(\theta)\ge A_\emptyset(\theta)\iff y\le y_n$ or $\Pr_{\theta,y}\left[A_y(\theta)\ge A_\emptyset(\theta)\right]>\frac{1}{2}\iff \Pr(y\le y_n)>\frac{1}{2}$. As we established above, this is equivalent to $y_n>\overline{\theta}$. □

Proof of Lemma 3

Proof. We show that

$$\mu_{y_n}>\theta_\emptyset^\star\implies \Delta\Phi\left(\theta_\emptyset^\star+k,\theta_\emptyset^\star;\mu_{y_n-\gamma}\right)>\Delta\Phi\left(\theta_\emptyset^\star,\theta_\emptyset^\star-k;\mu_{y_n+\gamma}\right)\tag{24}$$

$\forall\gamma\in\mathbb{R}_+$. The argument for $\mu_{y_n}\le\theta_\emptyset^*$ is analogous and omitted. Before we establish (24), we point out the following properties of the function $\Delta\Phi(\alpha,\beta;\mu)$:

Lemma 7. *$\Delta\Phi(\alpha,\beta;\mu)$ is a symmetric, single-peaked function of μ, with a global maximum at $\mu'=\frac{\alpha+\beta}{2}$.*

Proof. That $\Delta\Phi(\alpha,\beta;\mu)$ is single-peaked, with unique maximum at μ' follows immediately from after first- and second-order conditions with respect to μ. For symmetry, consider the following two values of μ: $\mu_1=\mu'+\Delta\mu$, $\mu_2=\mu'-\Delta\mu$, for some $\Delta\mu\in\mathbb{R}_+$. Then, $\Delta\Phi(\alpha,\beta;\mu_1)$ $=\Phi\left(\frac{\alpha-\mu'-\Delta\mu}{\sigma_y}\right)-\Phi\left(\frac{\beta-\mu'-\Delta\mu}{\sigma_y}\right)$ and $\Delta\Phi(\alpha,\beta;\mu_2)=\Phi\left(\frac{\alpha-\mu'+\Delta\mu}{\sigma_y}\right)-\Phi\left(\frac{\beta-\mu'+\Delta\mu}{\sigma_y}\right)$. Noting that $\alpha-\mu'=-(\beta-\mu')$ and using $\Phi(x)=1-\Phi(-x)$ establishes the result. □

Suppose that $\mu_{y_n} > \theta^\star_\emptyset$. We show here that $\left|\mu_{y_n+\gamma} - \left(\theta^*_\emptyset - \frac{k}{2}\right)\right| \geq \left|\mu_{y_n-\gamma} - \left(\theta^*_\emptyset + \frac{k}{2}\right)\right|$, $\forall \gamma, k \in \mathbb{R}_+$. Given Lemma 7, the conclusion of Lemma 3 will then immediately follow.

Since $\mu_{y_n} > \theta^*_\emptyset$, and $\mu_{y_n+\gamma}$ increases in γ, we have

$$\mu_{y_n+\gamma} - \theta^*_\emptyset + \frac{k}{2} > 0 \tag{25}$$

$\forall \gamma \in \mathbb{R}_+$. However, the difference

$$\mu_{y_n-\gamma} - \theta^*_\emptyset - \frac{k}{2} \tag{26}$$

can be positive or negative. If (26) is nonnegative, the claim follows trivially by inspection of (25) and (26). Thus, suppose $\mu_{y_n-\gamma} - \theta^*_\emptyset - \frac{k}{2} < 0$, or $\theta^*_\emptyset - \mu_{y_n} + \frac{k}{2} + \mu_{y_n} - \mu_{y_n-\gamma} \geq 0$. By the linearity of μ_y in y, $\mu_{y_n+\gamma} - \mu_{y_n} = \mu_{y_n} - \mu_{y_n-\gamma}$ and we can write (25) as

$$\begin{aligned} \mu_{y_n} - \theta^*_\emptyset + \frac{k}{2} + \mu_{y_n+\gamma} - \mu_{y_n} &= \left(\theta^*_\emptyset - \mu_{y_n} + \frac{k}{2} + \mu_{y_n} - \mu_{y_n-\gamma}\right) + 2\left(\mu_{y_n} - \theta^*_\emptyset\right) \\ &> \theta^*_\emptyset - \mu_{y_n} + \frac{k}{2} + \mu_{y_n} - \mu_{y_n-\gamma} \end{aligned}$$

where the inequality follows from $\mu_{y_n} > \theta^*_\emptyset$. □

Proof of Propositions 1 & 2

Proof. To establish both Propositions we first evaluate the derivative of y_n as $\bar{\theta}$ increases c correspondingly decreases to hold $\left(x^\star_\emptyset, \theta^\star_\emptyset\right)$ unchanged. In particular, we show below that $\frac{dy_n}{d\bar{\theta}} \mid_{\theta^*_\emptyset constant} \in (0,1)$, $\forall(\bar{\theta},\, c)$.

Total differentiation of (4) in $(\bar{\theta}, c)$ at $z = y_n$ gives

$$\begin{aligned} 0 &= (1-\omega_y)\left((1-\beta) + \beta\frac{dy_n}{d\bar{\theta}} \mid_{\theta^*_\emptyset}\right) - \frac{\omega_y^{\frac{1}{2}}}{\phi\left(\Phi^{-1}(c)\right)}\frac{dc}{d\bar{\theta}} \mid_{\theta^*_\emptyset} \\ &= \omega_\emptyset^{\frac{1}{2}}(1-\omega_y)\left((1-\beta) + \beta\frac{dy_n}{d\bar{\theta}} \mid_{\theta^*_\emptyset}\right) - (1-\omega_\emptyset)\,\omega_y^{\frac{1}{2}} \end{aligned} \tag{27}$$

where $\beta = \frac{\sigma^2_\theta}{\sigma^2_\theta + \sigma^2_\eta}$ and the second line follows after differentiation of (4) at $z = \emptyset$ and solving for $\frac{dc}{d\bar{\theta}} \mid_{\theta^*_\emptyset}$. Since $\omega_\emptyset > \omega_y$, it is clear by inspection that $\frac{dy_n}{d\bar{\theta}} \mid_{\theta^*_\emptyset} < 1$. Further, it can be verified by substitution that $\omega_\emptyset(1-\omega_y)(1-\beta) = \omega_y(1-\omega_\emptyset)$, allowing us to write (27) as

$$\omega_y^{-\frac{1}{2}} = \omega_\emptyset^{-\frac{1}{2}} + \omega_\emptyset^{-\frac{1}{2}}\beta\frac{dy_n}{d\bar{\theta}} \mid_{\theta^*_\emptyset}$$

Written this way, $\omega_\emptyset > \omega_y$ now implies that $\frac{dy_n}{d\bar{\theta}} \mid_{\theta^*_\emptyset} > 0$.

Since $\frac{dy_n}{d\bar{\theta}} |_{\theta^*_\emptyset constant} \in (0,1)$, we must both have that $y_n - \overline{\theta}$ is strictly decreasing and μ_{y_n} is strictly increasing in $\overline{\theta}$. Lemmas 2 and 3 then establish the existence of the appropriate thresholds $\lambda\left(\theta^*_\emptyset\right)$, $\delta\left(\theta^*_\emptyset\right)$ defined in Propositions 1 and 2, respectively.

We now show that and $\delta\left(\theta^*_\emptyset\right) < 0$ for $\theta^*_\emptyset > f\left(\frac{1}{2}\right)$.[45] To do that, we show that $y_n > \overline{\theta}$ when $\overline{\theta} = \theta^*_\emptyset > f\left(\frac{1}{2}\right)$. By Lemma 1, there is a $c < \frac{1}{2}$ which can support this as an equilibrium. Consider the function

$$g(\omega) = \omega x^*_\emptyset + (1-\omega)\overline{\theta} + \sigma_\epsilon \sqrt{\omega}\Phi^{-1}(c)$$

where $x^*_\emptyset$ is the equilibrium threshold under opacity, and $\sqrt{\omega}$ is the positive root of ω. Note that this function is of the same form as (22), but written for some arbitrary value of ω. We are interested in the properties of $g(\omega)$ on the domain $(0, \omega_\emptyset]$. In particular,

Lemma 8. *Assume $\theta^*_\emptyset = \bar{\theta} > \frac{1}{2}$. Then $g(\omega) < \theta^*_\emptyset$, $\forall \omega \in (0, \omega_\emptyset)$.*

Proof. Because $\left(x^*_\emptyset, \theta^*_\emptyset\right)$ is an equilibrium pair, it must satisfy equation (22) at $z = \emptyset$. But (22) can be written as $\theta^\star_\emptyset = g(\omega_\emptyset)$. Thus, $g(\omega_\emptyset) = \theta^*_\emptyset$. Further, it is easy to see that $g(0) = \bar{\theta}$. Since $\theta^*_\emptyset = \bar{\theta}$, we therefore have $g(0) = g(\omega_\emptyset) = \theta^*_\emptyset$.

Now, $g(\omega)$ is clearly *(i)* continuous on $[0, \omega_\emptyset]$; and *(ii)* strictly convex in ω. The latter follows because

$$g''(\omega) = \frac{\sigma_\epsilon}{4}\Phi^{-1}(c)\,\omega^{-\frac{3}{2}} > 0$$

where the inequality follows because $c < \frac{1}{2} \implies \Phi^{-1}(c) < 0$.

Since $g(0) = g(\omega_\emptyset)$ and g is strictly convex on $(0, \omega_\emptyset)$, it follows immediately from Jensen's inequality that $g(\omega) < \theta^\star_\emptyset$, $\forall \omega \in (0, \omega_\emptyset)$. □

Note that Lemma 8 applies to $\omega_y < \omega_\emptyset$, so that $g(\omega_y) < \theta^*_\emptyset$. However, since y_n satisfies $\left(x^*_{y=y_n} = x^*_\emptyset,\ \theta^*_{y=y_n} = \theta^*_\emptyset\right)$, equation (22) implies

$$\omega_y x^*_\emptyset + (1-\omega_y)\mu_{y_n} + \sigma_\epsilon\sqrt{\omega_y}\Phi^{-1}(c) = \theta^*_\emptyset \tag{28}$$

Together, $g(\omega_y) < \theta^*_\emptyset$ and (28) imply $\bar{\theta} < \mu_{y_n}$, and therefore $y_n > \overline{\theta}$ - as required. □

[45]The proof of the converse case is analogous, and therefore omitted.

Proof of Lemma 4

Proof. Denote for a variable d the differences $\Delta d_{-} := d^{*}_{y_n-\gamma} - d^{*}_{\emptyset}$, $\Delta d_{+} := d^{*}_{\emptyset} - d^{*}_{y_n+\gamma}$. Evaluating (22) at $z = y_n, y_n \pm \gamma$ and taking differences appropriately yields

$$\Delta\theta_{+} = \omega_y \Delta x_{+} + (1-\omega_y)\left(\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\eta^2}\right)\gamma \tag{29}$$

$$\Delta\theta_{-} = \omega_y \Delta x_{-} + (1-\omega_y)\left(\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\eta^2}\right)\gamma \tag{30}$$

Since $\omega_y > 0$, we can see by inspection of (29) and (30) that $\Delta\theta_{-} \geq \Delta\theta_{+} \iff \Delta x_{-} \geq \Delta x_{+}$. □

Proof of Lemma 5

Proof. Letting $\Delta\Phi(a,b,\mu) := \Phi\left(\frac{a-\mu}{\sigma_y}\right) - \Phi\left(\frac{b-\mu}{\sigma_y}\right)$, we define

$$T_{\mathcal{I}}(\rho) := \int_0^\infty \Delta\Phi\left(\theta^*_\emptyset, \theta^*_{y_n+\gamma}; \mu_{y_n+\gamma}\right) . \frac{(\phi(s(-\gamma)) - \phi(s(\gamma)))}{\sqrt{\sigma_\theta^2+\sigma_\eta^2}} . d\gamma \tag{31}$$

$$T_{\mathcal{T}}(\rho) := \int_0^\infty \left[\Delta\Phi\left(\theta^*_\emptyset + \Gamma_\gamma, \theta^*_\emptyset; \mu_{y_n-\gamma}\right) - \Delta\Phi\left(\theta^*_\emptyset, \theta^*_{y_n+\gamma}; \mu_{y_n+\gamma}\right)\right] \frac{\phi(s(-\gamma))}{\sqrt{\sigma_\theta^2+\sigma_\eta^2}} . d\gamma \tag{32}$$

$$T_{\mathcal{B}}(\rho) := \int_0^\infty \Delta\Phi\left(\theta^*_{y^n-\gamma}, \theta^*_\emptyset + \Gamma_\gamma; \mu_{y_n-\gamma}\right) \frac{\phi(s(-\gamma))}{\sqrt{\sigma_\theta^2+\sigma_\eta^2}} . d\gamma \tag{33}$$

where $\Gamma_\gamma := \theta^*_\emptyset - \theta^*_{y_n+\gamma}$ and $s(\gamma) = \frac{y_n+\gamma-\bar{\theta}}{\sqrt{\sigma_\theta^2+\sigma_\eta^2}}$. With these definitions and the substitution $y = y_n + \gamma$, it is easy to verify directly that $T(\rho) = T_{\mathcal{I}}(\rho) + T_{\mathcal{T}}(\rho) + T_{\mathcal{B}}(\rho)$.

We now show the following comparative statics: *(i)* $T_{\mathcal{I}}(\rho) \geq 0$ iff $A_{\bar{\theta}}(\theta) \geq A_\emptyset(\theta)$, $\forall\theta \in \mathbb{R}$; *(ii)* $T_{\mathcal{T}}(\rho) \geq 0$ iff (6) holds $\forall\gamma, k \in \mathbb{R}_+$; *(iii)* $T_{\mathcal{B}}(\rho) \geq 0$ if $\theta^\star_{y_n-\gamma} - \theta^\star_\emptyset \geq \theta^\star_\emptyset - \theta^\star_{y_n+\gamma}$, $\forall\gamma \in \mathbb{R}_+$.

The integrand in (31) evaluates the net *ex ante* impact of fundamentally opposite signals $y_n \pm \gamma$ on the chances of regime change, assuming that the impact of these fundamentally opposite signals on continuation outcomes is equal and opposite

$$\Pr\left(\theta^*_\emptyset \leq \theta \leq \theta'_{y_n-\gamma} \mid y_n - \gamma\right) = -\Pr\left(\theta^*_{y_n+\gamma} \leq \theta \leq \theta^*_\emptyset \mid y_n + \gamma\right)$$

where $\theta'_{y_n-\gamma}$ is a hypothetical threshold that equates the marginal continuation probabilities. Since we assume the conditional marginal impact of signals $y_n \pm \gamma$ is equal and opposite, their only ex ante difference is the probability that these signals are realized. We establish

Lemma 5.1 formally in the next Lemma:

Lemma 9. $T_I\left(\overline{\theta}, c, f\right) \geq \iff \Pr_{\theta,y}\left[A_y\left(\theta\right) \geq A_{\emptyset}\left(\theta\right)\right] > \frac{1}{2}$.

Proof. From single-peakedness and symmetry of ϕ, $T_{\mathcal{I}}\left(\overline{\theta}, c, f\right) \geq \iff y_n \geq \overline{\theta}$. Applying Lemma 2, the result follows. □

Notice that $T_{\mathcal{T}}\left(\overline{\theta}, c, f\right)$ can be expressed as a probability-weighted expectation of I's net resilience across opposite signals $y_n \pm \gamma$

$$\Pr\left(y \leq y_n\right).\mathbb{E}\left[\Delta\Phi\left(\theta^*_{\emptyset} + \Gamma_{\gamma}, \theta^*_{\emptyset}; \mu_{y_n-\gamma}\right) - \Delta\Phi\left(\theta^*_{\emptyset}, \theta^*_{y_n+\gamma}; \mu_{y_n+\gamma}\right) \mid y \leq y_n\right] \tag{34}$$

Thus, we have

Lemma 10. $T_{\mathcal{T}}\left(\overline{\theta}, c, f\right) \geq 0 \iff \Pr_{\theta}\left[\theta \in \left[\theta^*_{\emptyset}, \theta^*_{\emptyset} + k\right] \mid y_n - \gamma\right] > \Pr_{\theta}\left[\theta \in \left[\theta^*_{\emptyset} - k, \theta^*_{\emptyset}\right] \mid y_n + \gamma\right]$, $\forall k$, $\gamma \in \mathbb{R}_+$.

Proof. Let $k\left(\gamma\right) := \left|\theta^*_{\emptyset} - \theta^*_{y_n+\gamma}\right|$ for all $\gamma > 0$. Thus, if

$$\Pr_{\theta}\left[\theta \in \left[\theta^*_{\emptyset}, \theta^*_{\emptyset} + k\right] \mid y_n - \gamma\right] > \Pr_{\theta}\left[\theta \in \left[\theta^*_{\emptyset} - k, \theta^*_{\emptyset}\right] \mid y_n + \gamma\right]$$

holds for all k, γ, it must hold for all $k\left(\gamma\right) = \left|\theta^*_{\emptyset} - \theta^*_{y_n+\gamma}\right|$. Then this also holds in expectation, implying (34) is positive.

For the converse, if $T_{\mathcal{T}} > 0$, then from (34) there must be some γ for which

$$\Delta\Phi\left(\theta^*_{\emptyset} + \Gamma_{\gamma}, \theta^*_{\emptyset}; \mu_{y_n-\gamma}\right) > \Delta\Phi\left(\theta^*_{\emptyset}, \theta^*_{y_n+\gamma}; \mu_{y_n+\gamma}\right) \tag{35}$$

From Lemma 3, this is only the case when $\mu_{y_n} > \theta^*_{\emptyset}$. But Lemma 3 then implies that (35). holds for all $\gamma \in \mathbb{R}_+$. □

Finally $T_B\left(\overline{\theta}, c, f\right)$ can be expressed as follows

$$\Pr\left(y \leq y_n\right).\mathbb{E}\left[\Delta\Phi\left(\theta^*_{y^n-\gamma}, \theta^*_{\emptyset} + \Gamma_{\gamma}; \mu_{y_n-\gamma}\right) \mid y \leq y_n\right] \tag{36}$$

which is a probability-weighted expectation of the impact of transparency on equilibrium due to the excess impact of signal $y_n - \gamma$ over $y_n + \gamma$ on the threshold θ^*_y. Lemma 5.3 follows trivially from inspection of (36). □

Proof of Proposition 4

Proof. We establish the result in two steps. We first show that $\lim_{\sigma_\eta\to\infty} T(\rho) = 0$, $\forall c \in (0,1), \overline{\theta} \in \mathbb{R}, \sigma_\theta^2, \sigma_\epsilon^2 \in \mathbb{R}_{++}$. We then show that convergence happens from above if (11) holds. The converse argument is idential and therefore omitted.

Using the substitution $\gamma = \frac{y-\overline{\theta}}{\hat{\sigma}}$, where we use $\hat{\sigma}^2 = \sigma_\theta^2 + \sigma_\eta^2$ for ease of notation, we can write (10) as

$$T(\rho) = \int_{-\infty}^{\infty} \left[\Phi\left(\frac{\theta^\star_{\overline{\theta}+\hat{\sigma}\gamma} - \mu_{\overline{\theta}+\hat{\sigma}\gamma}}{\sigma_y} \right) - \Phi\left(\frac{\theta^\star_{\emptyset} - \mu_{\overline{\theta}+\hat{\sigma}\gamma}}{\sigma_y} \right) \right] \phi(\gamma)\, d\gamma \tag{37}$$

where $\left(x^\star_{\overline{\theta}+\hat{\sigma}\gamma}, \theta^\star_{\overline{\theta}+\hat{\sigma}\gamma}\right)$ solves (4)-(5) for $z = \overline{\theta} + \hat{\sigma}\gamma$ and recall $\sigma_y^2 := var(\theta \mid y) = \frac{\sigma_\theta^2(\hat{\sigma}^2 - \sigma_\theta^2)}{\hat{\sigma}^2}$. Since Φ is a standard normal cdf, the function $h(\gamma, \hat{\sigma}) := \Phi\left(\frac{\theta^\star_{\overline{\theta}+\hat{\sigma}\gamma} - \mu_{\overline{\theta}+\hat{\sigma}\gamma}}{\sigma_y}\right) - \Phi\left(\frac{\theta^\star_{\emptyset} - \mu_{\overline{\theta}+\hat{\sigma}\gamma}}{\sigma_y}\right)$ is uniformly bounded above by the (integrable) function $g(\gamma) = 1$. Thus, we can apply the Dominated Convergence Theorem to (37), which yields the limiting expression $\lim_{\sigma_\eta^2\to\infty} T(\rho) = \int_{-\infty}^{\infty} \left(\lim_{\sigma_\eta^2\to\infty} h(\gamma,\hat{\sigma})\right) \phi(\gamma)\, d\gamma$. Since *(i)* $(x^\star_z, \theta^\star_z)$ uniquely solves (4)-(5) $\forall z \in Z$, *(ii)* is continuous in ω and *(iii)* $\omega_y \to \omega_\emptyset$, $\mu_y \to \overline{\theta}$ as $\sigma_\eta^2 \to \infty$, $\lim_{\sigma_\eta^2\to\infty}\left(x^\star_{\overline{\theta}+\hat{\sigma}\gamma}, \theta^\star_{\overline{\theta}+\hat{\sigma}\gamma}\right)$ is well-defined for all $\gamma \in \mathbb{R}$ and equal to $\left(x^\star_\emptyset, \theta^\star_\emptyset\right)$. Therefore by the continuity of Φ, $\lim_{\sigma_\eta^2\to\infty} h(\gamma,\hat{\sigma}) = 0$, $\forall \gamma \in \mathbb{R}$. Thus, $\lim_{\sigma_\eta^2\to\infty} T(\rho) = 0$ as required.

We now derive necessary and sufficient conditions under which $T(\rho)$ converges to 0 from above. To do this, we establish limiting properties of the derivative, $\frac{\partial T}{\partial \sigma_\eta}(\rho)$. In particular, if we can find a positive-valued scaling factor $s(\hat{\sigma}) > 0$, $\forall \hat{\sigma} \geq 0$, such that $\lim_{\sigma_\eta\to\infty} s(\hat{\sigma}) \frac{\partial T}{\partial \sigma_\eta}(\rho) < 0$, then $\lim_{\sigma_\eta\to\infty} T(\rho) = 0^+$. We now show that such a scaling factor exists if (11) holds.

We take the partial derivative of $T(\rho)$ in $\hat{\sigma}$, holding $c, \overline{\theta}, \sigma_\epsilon^2, \sigma_\theta^2$ fixed.[46] From (10), $\frac{\partial T}{\partial \hat{\sigma}} = \frac{\partial \Pr\left(\theta \leq \theta_y^\star\right)}{\partial \hat{\sigma}}$, which follows since $\Pr\left(\theta \leq \theta^\star_\emptyset\right)$ does not depend on $\hat{\sigma}$. Again using the substitution $\gamma = \frac{y-\overline{\theta}}{\hat{\sigma}}$ and noting that $\theta^\star_{\overline{\theta}+\hat{\sigma}\gamma}$ is a differentiable function of $\hat{\sigma}, \gamma$, we can express the derivative as

$$\frac{\partial T}{\partial \hat{\sigma}} = \frac{\left(\hat{\sigma}^2 - \sigma_\theta^2\right)^{-\frac{1}{2}}}{\hat{\sigma}^2 \sigma_\theta} \int_{-\infty}^{\infty} \left[\gamma \sigma_\theta^2 \hat{\sigma} - \frac{\sigma_\theta^2 \hat{\sigma}^2}{\hat{\sigma}^2 - \sigma_\theta^2} \left(\theta_y^\star - \overline{\theta} - \gamma \sigma_\theta^2 \hat{\sigma}^{-1} \right) + \hat{\sigma}^3 \frac{\partial \theta_y^\star}{\partial \hat{\sigma}} \right] \phi(\cdot)\, \phi(\gamma)\, d\gamma \tag{38}$$

where $\phi(\cdot) = \phi\left(\frac{\theta_y^\star - \mu_y}{\sigma_y}\right)$ and $y = \overline{\theta} + \hat{\sigma}\gamma$. Totally differentiating system (4)-(5), we can

[46] Given σ_θ^2 fixed, $\frac{\partial T}{\partial \sigma_\eta^2} = \frac{1}{2}\hat{\sigma}^{-1} \frac{\partial T}{\partial \hat{\sigma}}$. Since $\hat{\sigma} > 0$, these derivatives always have the same sign.

calculate $\frac{\partial\theta^\star_{\overline{\theta}+\hat{\sigma}\gamma}}{\partial\hat{\sigma}}$:

$$\frac{\partial\theta^\star_{\overline{\theta}+\hat{\sigma}\gamma}}{\partial\hat{\sigma}} = \frac{\left(x^\star_{\overline{\theta}+\hat{\sigma}\gamma} - \overline{\theta} - \sigma_\theta^2\gamma\hat{\sigma}^{-1} + \frac{\sigma_\epsilon}{2}\omega_y^{-\frac{1}{2}}\Phi^{-1}(c)\right)}{1-\omega_y v\left(x^\star_{\overline{\theta}+\hat{\sigma}\gamma}, \theta^\star_{\overline{\theta}+\hat{\sigma}\gamma}\right)}\frac{\partial\omega_y}{\partial\hat{\sigma}} - \frac{(1-\omega_y)\,\sigma_\theta^2\gamma\hat{\sigma}^{-2}}{1-\omega_y v\left(x^\star_{\overline{\theta}+\hat{\sigma}\gamma}, \theta^\star_{\overline{\theta}+\hat{\sigma}\gamma}\right)} \tag{39}$$

where $v(x,\theta) := \sigma_\epsilon\left(f'\left(\Phi\left(\frac{x-\theta}{\sigma_\epsilon}\right)\right)\phi\left(\frac{x-\theta}{\sigma_\epsilon}\right)\right)^{-1} + 1$ and $\frac{\partial\omega_y}{\partial\hat{\sigma}} = \frac{2\sigma_\theta^4\sigma_\epsilon^2\hat{\sigma}}{\left(\sigma_\theta^2(\hat{\sigma}^2-\sigma_\epsilon^2)+\hat{\sigma}^2\sigma_\epsilon^2\right)^2}$. Substituting (39) into (38) yields, on rearrangement

$$\begin{aligned} s(\hat{\sigma})\frac{\partial T}{\partial\hat{\sigma}} &= \int_{-\infty}^{\infty}\left(1+\frac{1-\omega_y}{\omega_y v\left(x^\star_y,\theta^\star_y\right)-1}\right)\sigma_\theta^2\hat{\sigma}\gamma\phi(\cdot)\,\phi(\gamma)\,d\gamma \\ &\quad - \int_{-\infty}^{\infty}\frac{\left(x^\star_y - \overline{\theta} - \sigma_\theta^2\gamma\hat{\sigma}^{-1} + \frac{\sigma_\epsilon}{2}\omega_y^{-\frac{1}{2}}\Phi^{-1}(c)\right)}{\omega_y v\left(x^\star_y,\theta^\star_y\right)-1}\hat{\sigma}^3\frac{\partial\omega_y}{\partial\hat{\sigma}}\phi(\cdot)\,\phi(\gamma)\,d\gamma \\ &\quad - \frac{\sigma_\theta^2\hat{\sigma}^2}{\hat{\sigma}^2-\sigma_\theta^2}\int_{-\infty}^{\infty}\left(\theta^\star_y - \overline{\theta} - \gamma\sigma_\theta^2\hat{\sigma}^{-1}\right)\phi(\cdot)\,\phi(\gamma)\,d\gamma \end{aligned} \tag{40}$$

where $s(\hat{\sigma}) = \sigma_\theta\hat{\sigma}^2\left(\hat{\sigma}^2-\sigma_\theta^2\right)^{\frac{1}{2}} > 0$. We now establish the limiting properties of each of the three integrals in (40) as $\hat{\sigma}\to\infty$. Consider the third integral in (40). The expression $\left(\theta^\star_y - \overline{\theta} - \gamma\sigma_\theta^2\hat{\sigma}^{-1}\right)\phi(\cdot)$ is bounded in $\gamma,\hat{\sigma}$ by $g_1(\gamma) := \left(\sqrt{2\pi}\right)^{-1}\left(\max\left\{1-\overline{\theta},\overline{\theta}\right\} + \sigma_\theta^2|\gamma|\right)$, for all $\hat{\sigma}\geq 1$. Moreover, g_1 has a finite expectation when γ follows a standard normal distribution. Therefore, we can apply the Dominated Convergence Theorem to establish:

$$\begin{aligned} \lim_{\hat{\sigma}\to\infty}\frac{\sigma_\theta^2\hat{\sigma}^2}{\hat{\sigma}^2-\sigma_\theta^2}\int_{-\infty}^{\infty}\left(\theta^\star_y - \overline{\theta} - \gamma\sigma_\theta^2\hat{\sigma}^{-1}\right)\phi(\cdot)\,\phi(\gamma)\,d\gamma &= \sigma_\theta^2\int_{-\infty}^{\infty}\left[\lim_{\hat{\sigma}\to\infty}\left(\theta^\star_y - \overline{\theta} - \gamma\sigma_\theta^2\hat{\sigma}^{-1}\right)\phi(\cdot)\right]\phi(\gamma)\,d\gamma \\ &= \sigma_\theta^2\left(\theta^\star_\emptyset - \overline{\theta}\right)\phi\left(\frac{\theta^\star_\emptyset - \overline{\theta}}{\sigma_\theta}\right) \end{aligned} \tag{41}$$

Next, consider the second integral in (40). Under assumption (2), it is easy to show that $\omega_y v\left(x^\star_y,\theta^\star_y\right) - 1 \geq \omega_\emptyset v\left(\frac{1}{2},\frac{1}{2}\right) - 1 > 0$, for all y. Now consider the function $r(x,\omega_y) := \frac{x-\overline{\theta}+\frac{\sigma_\epsilon}{2}\omega_y^{-\frac{1}{2}}\Phi^{-1}(c)}{\omega_y v(x,\theta_P(x))-1}$. $r(x,\omega)$ is continuous in x, ω and satisfies $\lim_{x\to\pm\infty} r(x,\omega) = 0$. Therefore, this function has a finite upper (lower) bound $\overline{r}(\omega)$, $(\underline{r}(\omega))$ achieved at some interior value of x which independent of γ (γ does not appear in $r(x,\omega)$). Since these bounds are finite for all ω that obey (2), they must be finite for $\omega = \omega_\emptyset$.

We now show that for all $\hat{\sigma}$ sufficiently high that (2) holds, the solutions to the problems $\max_{x\in\mathbb{R}} r(x,\omega_y)$ (resp. $\min_{x\in\mathbb{R}} r(x,\omega_y)$) are contained in a compact subinterval of $\mathbb{R}$, bounded independently of ω_y. In particular, for any $\delta > 0$ such that (2) holds on $\omega \in [\omega_\emptyset - \delta, \omega_\emptyset]$, we can bound $|r(x,\omega)| \leq \max_{i\in\{0,1\}} \frac{\left|x-\overline{\theta}+\frac{\sigma_\epsilon}{2}(\omega_\emptyset - i\delta)^{-\frac{1}{2}}\Phi^{-1}(c)\right|}{(\omega_\emptyset-\delta)v(x,\theta_P(x))-1} := \tilde{r}(x,\delta)$.[47] It is easy to verify that $\lim_{x\to\pm\infty} \tilde{r}(x,\delta) = 0$. Accordingly, there must exist a B such that $\tilde{r}(x,\delta) < \frac{1}{2}\max\{|\overline{r}(\omega_\emptyset)|, |\underline{r}(\omega_\emptyset)|\}$ for all $|x| \geq B$ - This B is the uniform bound on x we required. Selecting $\hat{\sigma}$ sufficiently high that $\omega_y \in [\omega_\emptyset - \delta, \omega_\emptyset]$ establishes the claim.

Given continuity of $r(x,\omega)$ in x, ω and the compactness of interval $[-B, B]$, we can apply the Theorem of the Maximum. Therefore, $\lim_{\omega_y\to\omega_\emptyset} \overline{r}(\omega_y) = \overline{r}(\omega_\emptyset)$. Since $\omega_y \to \omega_\emptyset$ as $\hat{\sigma} \to \infty$, there exists some $\hat{\sigma}_N$ such that $r(x)$ is uniformly bounded for all $\hat{\sigma} \geq \hat{\sigma}_N$.

Notice that the second integral in (40) can be written in terms of $r\left(x_y^\star\right)$. In particular, for all $\hat{\sigma} \geq \max\{\hat{\sigma}_N, 1\}$ the function $\left(r\left(x_y^\star\right) - \frac{\sigma_\theta^2\gamma\hat{\sigma}^{-1}}{\omega_y v\left(x_y^\star,\theta_y^\star\right)-1}\right).\phi(\cdot)$ is bounded by $g_2(\gamma) := \left(\sqrt{2\pi}\right)^{-1}\left(\overline{r}(\omega_\emptyset) + \epsilon + \frac{\sigma_\theta^2}{\omega_\emptyset v\left(\frac{1}{2},\frac{1}{2}\right)-1}|\gamma|\right)$, where $\mathbb{E}[g_2(\gamma)] < \infty$. Applying the Dominated Convergence Theorem and noticing that $\lim_{\hat{\sigma}\to\infty} \hat{\sigma}^3\frac{\partial\omega_y}{\partial\hat{\sigma}} = 2\sigma_\theta^2\omega_\emptyset(1-\omega_\emptyset)$, $r\left(x_\emptyset^\star, \omega_\emptyset\right) = -\frac{\partial\theta_\theta^\star}{\partial\omega_\emptyset}$ we have:

$$\lim_{\hat{\sigma}\to\infty} \hat{\sigma}^3\frac{\partial\omega_y}{\partial\hat{\sigma}}\int_{-\infty}^{\infty}\left(r\left(x_y^\star\right) - \frac{\sigma_\theta^2\gamma\hat{\sigma}^{-1}}{\omega_y v\left(x_y^\star,\theta_y^\star\right)-1}\right)\phi(\cdot)\,\phi(\gamma)\,d\gamma = -2\sigma_\theta^2\omega_\emptyset(1-\omega_\emptyset)\frac{\partial\theta_\theta^\star}{\partial\omega_\emptyset}\phi\left(\frac{\theta_\emptyset^\star-\overline{\theta}}{\sigma_\theta}\right) \tag{42}$$

Finally, consider the first integral in (40). By symmetry of $\phi(\gamma)$, this can be rewritten

$$\sigma_\theta^2\int_0^\infty \hat{\sigma}\gamma\phi(\gamma)\left[t\left(\hat{\sigma}, x_{\overline{\theta}+\hat{\sigma}\gamma}^\star, \theta_{\overline{\theta}+\hat{\sigma}\gamma}^\star\right)\phi\left(\frac{\theta_{\overline{\theta}+\hat{\sigma}\gamma}^\star-\mu_{\overline{\theta}+\hat{\sigma}\gamma}}{\sigma_y}\right) - t\left(\hat{\sigma}, x_{\overline{\theta}-\hat{\sigma}\gamma}^\star, \theta_{\overline{\theta}-\hat{\sigma}\gamma}^\star\right)\phi\left(\frac{\theta_{\overline{\theta}-\hat{\sigma}\gamma}^\star-\mu_{\overline{\theta}-\hat{\sigma}\gamma}}{\sigma_y}\right)\right]d\gamma \tag{43}$$

where $t(\hat{\sigma},x,\theta) = 1 + \frac{1-\omega_y}{\omega_y v(x,\theta)-1}$. Under assumption (2), $1 \leq t(\hat{\sigma},x,\theta) \leq \overline{t} := 1 + \frac{1-\omega_\emptyset+\delta}{(\omega_\emptyset-\delta)v\left(\frac{1}{2},\frac{1}{2}\right)-1} < \infty$ for all $\hat{\sigma}$ such that $\omega_y \in [\omega_\emptyset - \delta, \omega_\emptyset]$, $x, \theta \in \mathbb{R}$. Denoting $t_\pm(\gamma,\hat{\sigma}) := t\left(\hat{\sigma}, x_{\overline{\theta}\pm\hat{\sigma}\gamma}^\star, \theta_{\overline{\theta}\pm\hat{\sigma}\gamma}^\star\right)$ and $\phi_\pm(\gamma,\hat{\sigma}) := \phi\left(\frac{\theta_{\overline{\theta}+\hat{\sigma}\gamma}^\star-\mu_{\overline{\theta}+\hat{\sigma}\gamma}}{\sigma_y}\right)$ to ease notation, the term in square brackets in (43) can be bounded using the triangle inequality

$$|t_+\phi_+ - t_-\phi_-| \;\leq\; \overline{t}\,|\phi_+ - \phi_-| + |t_+ - t_-|\,\phi_-$$

Noting that the standard normal p.d.f. has bounded derivative, $\phi'(z) \leq \phi'_{\max} := \frac{e^{-1}}{\sqrt{2\pi}}$, we

[47] Such a $\delta > 0$ exists, since (2) holds with inequality at $\omega_\emptyset$.

can further bound the first term on the RHS as follows

$$\begin{aligned}\bar{t}\left|\phi_{+}-\phi_{-}\right| &\leq \bar{t}\phi'_{\max}\cdot\left|\left(\frac{\hat{\sigma}}{\sigma_{\theta}\left(\hat{\sigma}^{2}-\sigma_{\theta}^{2}\right)^{\frac{1}{2}}}\right)\left(\theta^{\star}_{\overline{\theta}+\hat{\sigma}\gamma}-\mu_{\overline{\theta}+\hat{\sigma}\gamma}-\left(\theta^{\star}_{\overline{\theta}-\hat{\sigma}\gamma}-\mu_{\overline{\theta}-\hat{\sigma}\gamma}\right)\right)\right| \\ &= \bar{t}\phi'_{\max}\frac{\hat{\sigma}}{\sigma_{\theta}}\left(\hat{\sigma}^{2}-\sigma_{\theta}^{2}\right)^{-\frac{1}{2}}\left|\theta^{\star}_{\overline{\theta}+\hat{\sigma}\gamma}-\theta^{\star}_{\overline{\theta}-\hat{\sigma}\gamma}-2\frac{\sigma_{\theta}\gamma}{\hat{\sigma}}\right| \\ &\leq 2\bar{t}\phi'_{\max}\frac{\hat{\sigma}}{\sigma_{\theta}}\left(\hat{\sigma}^{2}-\sigma_{\theta}^{2}\right)^{-\frac{1}{2}}\left|\frac{1-\omega_{y}}{\omega_{y}v\left(\frac{1}{2},\frac{1}{2}\right)-1}+1\right|\frac{\sigma_{\theta}\gamma}{\hat{\sigma}}\end{aligned}$$

where the second inequality follows because $\left|\theta^{\star}_{\overline{\theta}+\hat{\sigma}\gamma}-\theta^{\star}_{\overline{\theta}-\hat{\sigma}\gamma}\right| \leq (\sup\frac{\partial\theta^{\star}_{y}}{\partial\mu_{y}}).\left|\mu_{\overline{\theta}+\hat{\sigma}\gamma}-\mu_{\overline{\theta}-\hat{\sigma}\gamma}\right|$, and differentiation of (4)-(5) in μ_y yields $\frac{\partial\theta^{\star}_{y}}{\partial\mu_{y}}=\frac{1-\omega_{y}}{\omega_{y}v\left(x^{\star}_{\emptyset},\theta^{\star}_{\emptyset}\right)-1}\leq\frac{1-\omega_{y}}{\omega_{y}v\left(\frac{1}{2},\frac{1}{2}\right)-1}$.

We can bound $|t_{+}-t_{-}|$ in a similar way. Taking derivatives,

$$\frac{\partial t}{\partial\mu_{y}}=-\frac{\left(1-\omega_{y}\right)^{2}\omega_{y}}{\left(\omega_{y}\sigma_{\epsilon}-\left(1-\omega_{y}\right)f'\left(\cdot\right)\phi\left(\cdot\right)\right)^{2}}\frac{v\left(x^{\star},\theta^{\star}\right)-1}{\omega_{y}v\left(x^{\star},\theta^{\star}\right)-1}\left(f''\left(\cdot\right)\left(\phi\left(\cdot\right)\right)^{2}+f'\phi'\left(\cdot\right)\right)$$

where ϕ, ϕ' are evaluated at $\frac{x-\theta}{\sigma_{\epsilon}}$ and f', f'' at $\Phi\left(\frac{x-\theta}{\sigma_{\epsilon}}\right)$. Since f', f'' are bounded and continuous, $\frac{\partial t}{\partial\mu_{y}}$ is continuous in $x^{\star}$ (after writing $\theta^{\star}=\theta_{P}\left(x^{\star}\right)$) and satisfies $\lim_{x^{\star}\to\pm\infty}\frac{\partial t}{\partial\mu_{y}}=0$ so long as assumption (2) holds. Thus, $\frac{\partial t}{\partial\mu_{y}}$ attains an interior maximum in x, for any $\hat{\sigma}$. Note that the value of x that maximizes this expression depends only on ω_y. Let $\tau\left(\omega\right)=\max_{x^{\star}\in\mathbb{R}}\frac{\partial t}{\partial\mu_{y}}$.[48] As $\omega_{y}\to\omega_{\emptyset}$, the Theorem of the Maximum implies that $\tau\left(\omega_{y}\right)\to\tau\left(\omega_{\emptyset}\right)<\infty$. Thus for any $\epsilon>0$, there is a $\hat{\sigma}_{M}$ such that $\frac{\partial t}{\partial\mu_{y}}$ is bounded above by $\tau\left(\omega\right)+\epsilon$, for all $\hat{\sigma}\geq\hat{\sigma}_{M}$. For all such $\hat{\sigma}_{M}$, we can apply the same argument above to establish that $|t_{+}-t_{-}|\leq 2\left(\tau\left(\omega\right)+\epsilon\right)\frac{\sigma_{\theta}\gamma}{\hat{\sigma}}$. Taken together, we have established the existence of a constant $\infty>C>0$ such that for all $\hat{\sigma}$ large enough, the function $\gamma\hat{\sigma}\left(t_{+}\phi_{+}-t_{-}\phi_{-}\right)$ is bounded by the integrable function $C\gamma^{2}$. Thus, we can apply the Dominated Convergence Theorem to (43).

Let $\xi:=\hat{\sigma}^{-1}$ and $\tilde{t}\left(\xi,x,\theta\right):=t\left(\xi^{-1},x,\theta\right)$. Then the integrand in (43) can be written $I\left(\xi,\gamma\right)\gamma\phi\left(\gamma\right)$, where $I\left(\xi,\gamma\right):=\left[\frac{\tilde{t}\left(\xi,x^{\star}_{\overline{\theta}+\xi^{-1}\gamma},\theta^{\star}_{\overline{\theta}+\xi^{-1}\gamma}\right)\phi_{+}-\tilde{t}\left(\xi,x^{\star}_{\overline{\theta}-\xi^{-1}\gamma},\theta^{\star}_{\overline{\theta}-\xi^{-1}\gamma}\right)\phi_{-}}{\xi}\right]$. As $\xi\to 0$, both the numerator and denominator of this expression converge to 0. Applying l'Hopital's rule

$$\lim_{\xi\to 0}I\left(\xi,\gamma\right)=2\frac{\partial\tilde{t}}{\partial x}\left(\frac{\partial x^{\star}_{y}}{\partial\xi}-\frac{\partial\theta^{\star}_{y}}{\partial\xi}\right)\phi\left(\frac{\theta^{\star}_{\emptyset}-\overline{\theta}}{\sigma_{\theta}}\right)+\frac{2}{\sigma_{\theta}}\phi'\left(\frac{\theta^{\star}_{\emptyset}-\overline{\theta}}{\sigma_{\theta}}\right)\left(\frac{\partial\theta^{\star}_{y}}{\partial\xi}-\frac{\partial\mu_{y}}{\partial\xi}\right)t\left(0,x^{\star}_{\emptyset},\theta^{\star}_{\emptyset}\right)$$

[48] Using an argument very similar to that used for $r\left(x,\omega\right)$, one can show that τ obeys the conditions to apply the Theorem of the Maximum. In particular, the $x^{\star}$ values solving $\max\frac{\partial t}{\partial\mu}$ live in a compact subinterval of $\mathbb{R}$. We omit the proof for the sake of brevity.

where all functions are evaluated at $\xi = 0$, $x^\star_{\overline{\theta}\pm\xi^{-1}\gamma} = x^\star_\emptyset$, $\theta^\star_{\overline{\theta}\pm\xi^{-1}\gamma} = \theta^\star_\emptyset$, and where we have used the following observations: *(i)* $\frac{\partial \tilde{t}}{\partial x} = -\frac{\partial \tilde{t}}{\partial \theta}$, and *(ii)* $\frac{\partial x^\star_{\overline{\theta}+\xi^{-1}\gamma}}{\partial \xi} = -\frac{\partial x^\star_{\overline{\theta}-\xi^{-1}\gamma}}{\partial \xi}$, *(iii)* $\frac{\partial \theta^\star_{\overline{\theta}+\xi^{-1}\gamma}}{\partial \xi} = -\frac{\partial \theta^\star_{\overline{\theta}-\xi^{-1}\gamma}}{\partial \xi}$ when $x^\star_{\overline{\theta}\pm\xi^{-1}\gamma} = x^\star_\emptyset$, $\theta^\star_{\overline{\theta}\pm\xi^{-1}\gamma} = \theta^\star_\emptyset$.[49]

Direct computation of these derivatives yields, as $\xi \to 0$

$$-\frac{\partial \tilde{t}}{\partial \theta^\star}\left(\frac{\partial x^\star_y}{\partial \xi} - \frac{\partial \theta^\star_y}{\partial \xi}\right) = \frac{\partial \tilde{t}}{\partial \theta^\star}(v-1)\frac{\partial \theta^\star}{\partial \xi} = \sigma^2_\theta (v-1)\frac{\partial \tilde{t}}{\partial x}\frac{\partial \theta^\star}{\partial \mu}\gamma$$

$$\sigma_\theta^{-1}\phi'\left(\frac{\theta^\star_\emptyset - \overline{\theta}}{\sigma_\theta}\right)\left(\frac{\partial \theta^\star_y}{\partial \xi} - \frac{\partial \mu_y}{\partial \xi}\right) t\left(0, x^\star_\emptyset, \theta^\star_\emptyset\right) = t\left(0, x^\star_\emptyset, \theta^\star_\emptyset\right)^2 \left(\theta^\star_\emptyset - \overline{\theta}\right)\phi\left(\frac{\theta^\star_\emptyset - \overline{\theta}}{\sigma_\epsilon}\right)\gamma$$

By the Dominated Convergence Theorem as $\hat{\sigma} \to \infty$, (43) limits to $\sigma^2_\theta \int_0^\infty (\lim_{\xi\to 0} I(\xi,\gamma))\gamma\phi(\gamma)\,d\gamma$. Noticing that $\lim_{\xi\to 0} I(\xi,\gamma)$ can be written $I(\xi)\gamma$, (43) becomes $\sigma^2_\theta I(\xi)\int_0^\infty \gamma^2\phi(\gamma)\,d\gamma = \sigma^2_\theta \frac{I(\xi)}{2}$ where[50]

$$\frac{I(\xi)}{2} \quad = \quad \phi\left(\frac{\theta^\star_\emptyset - \overline{\theta}}{\sigma_\theta}\right)\left[\sigma^2_\theta (v-1)\frac{\partial \tilde{t}}{\partial x}\frac{\partial \theta^\star}{\partial \mu} + t\left(0, x^\star_\emptyset, \theta^\star_\emptyset\right)^2\left(\theta^\star_\emptyset - \overline{\theta}\right)\right] \tag{44}$$

Plugging equations (41), (42) and (44) into (40), we can see on rearrangement that $T(\rho)$ approaches 0 from above iff

$$\left(2 - \frac{\partial \theta^\star_\emptyset}{\partial \mu}\right)\left(-\frac{\partial \theta^\star_\emptyset}{\partial \mu}\right)\left(\overline{\theta} - \theta^\star_\emptyset\right) - 2\omega_\emptyset(1-\omega_\emptyset)\frac{\partial \theta^\star_\emptyset}{\partial \omega_\emptyset} - \sigma^2_\theta(v-1)\frac{\partial t}{\partial \theta^\star_\emptyset}\frac{\partial \theta^\star_\emptyset}{\partial \mu} > 0$$

The interpretation of each term is discussed in the main text.

An identical argument shows that $\lim_{\sigma_\eta\to\infty} T(\rho) = 0^-$ when (11) is violated, establishing necessity and sufficiency.

□

Proof of Proposition 5

Proof. We show the result for $\overline{\theta} \to -\infty$. The proof for the converse case is analogous and therefore omitted. For any C^2 function f, (4) can be rewritten using the equilibrium

[49] The first observation is clear on inspection of function t. The second and third follow from differentiation of (4)-(5), after substituting $\delta = \hat{\sigma}^{-1}$.

[50] $\int_0^\infty \gamma^2\phi(\gamma)\,d\gamma = \frac{1}{2}$, since ϕ is a standard normal density.

conditions (4) and (5) as

$$
\begin{aligned}
0 > \quad & \frac{\sigma_\theta^2}{\sigma_\epsilon}\omega_\emptyset\left(1-\omega_\emptyset\right)\left(\frac{v\left(x_\emptyset^\star,\theta_\emptyset^\star\right)-1}{\omega_\emptyset v\left(x_\emptyset^\star,\theta_\emptyset^\star\right)-1}\right)^2\left(\frac{f''(\cdot)}{f'(\cdot)}\phi\left(\frac{x_\emptyset^\star-\theta_\emptyset^\star}{\sigma_\epsilon}\right)+\frac{x_\emptyset^\star-\theta_\emptyset^\star}{\sigma_\epsilon}\right)\\
+ \quad & \left(\frac{1-\omega_\emptyset}{\omega_\emptyset v\left(x_\emptyset^\star,\theta_\emptyset^\star\right)-1}\right)\left(\theta_\emptyset^\star-\overline{\theta}\right)+\sigma_\epsilon\omega_\emptyset^{\frac{1}{2}}\Phi^{-1}(c)
\end{aligned} \tag{45}
$$

where f', f'' are evaluated at $\Phi\left(\frac{x_\emptyset^\star-\theta_\emptyset^\star}{\sigma_\epsilon}\right)$. As $\overline{\theta}\to-\infty$, it is easy to verify from (5) that $x_\emptyset^\star\to\infty$. However, $\theta_\emptyset^\star$ always satisfies $\theta_\emptyset^\star\leq f(1)$. Therefore, $x_\emptyset^\star-\theta_\emptyset^\star\to\infty$ and $\phi\left(\frac{x_\emptyset^\star-\theta_\emptyset^\star}{\sigma_\epsilon}\right)\to\infty$. Similarly, we can calculate $\lim_{\overline{\theta}\to-\infty}\left(\frac{v\left(x_\emptyset^\star,\theta_\emptyset^\star\right)-1}{\omega_\emptyset v\left(x_\emptyset^\star,\theta_\emptyset^\star\right)-1}\right)^2=\frac{1}{\omega_\emptyset^2}$. For C^2 functions on a compact domain, $\frac{f''}{f'}$ is bounded and therefore, the first expression in (45) converges to $\frac{\sigma_\theta^2}{\sigma_\epsilon^2}\frac{(1-\omega_\emptyset)}{\omega_\emptyset}\left(x_\emptyset^\star-\theta_\emptyset^\star\right)\to\infty$.

The second term can be shown to converge to 0 as $\overline{\theta}\to-\infty$. [51] Finally, the third term is a constant in c. Thus, the expression in (45) has the same sign as $T_{\mathcal{B}}$ for sufficiently large $\overline{\theta}$. □

Proof of Proposition 6

Proof. As $\tilde{\sigma}^2\to\infty$, the solution curve $\theta_P(x)$ to (17) limits pointwise, for all $x\neq\overline{\theta}$, to

$$
\theta_P^l(x)=\begin{cases} f(1), & if\, x>\overline{\theta}\\ f(0) & ,if\, x<\overline{\theta}\end{cases} \tag{46}
$$

which follows since $\tilde{\sigma}^2,\rho_z,k_z\to 0$, $\forall z\in Z$ and $\sigma_\epsilon^2\leq p\tilde{\sigma}^2\to 0$. By contrast, the solution curve to (18), $\theta_z(x)$, limits to

$$
\theta_z^l(x)=\nu^l x+\rho_z\hat{y}-\left(1-\nu^l-\rho_z\right)\overline{\theta}+\left(\nu^l\sigma_\epsilon^2+(1-\rho_z)\hat{\sigma}^2\right)\Phi^{-1}(c) \tag{47}
$$

where $\nu^l:=\lim_{\tilde{\sigma}\to\infty}\nu$ satisfies $1\geq\nu^l\geq\frac{1}{1+p}$. Since $\theta_z^l(x)$ is strictly increasing in x, the limiting system (46)-(47) clearly always has at least one solution at z satisfying $x_z^{\star,l}<\overline{\theta}$ or $x_z^{\star,l}>\overline{\theta}$.[52] Moreover, these solutions must be limiting equilibria of (17)-(18) as $\tilde{\sigma}^2\to\infty$, $\nu\to\nu^l$: since $x_z^{\star,l}\neq\overline{\theta}$, we can find an $\epsilon>0$ such that $\theta_P(x)$, $\theta_P^l(x)$, $\theta_z(x)$ and $\theta_z^l(x)$ are continuous on the compact set $x\in\left[x_z^{\star,l}-\epsilon,x_z^{\star,l}+\epsilon\right]$, for all $\tilde{\sigma}$. Thus, convergence of

[51] Proof available on request. We omit the argument since the second term clearly takes the sign of the first in the limit.

[52] There can in general be at most three solutions to this system: one of each satisfying $x^\star<\overline{\theta}$, $x^\star=\overline{\theta}$ and $x^\star>\overline{\theta}$. If $\theta_z^l\left(\overline{\theta}\right)>(<)f(1)$, there is a unique solution, satisfying $x^\star<(>)\overline{\theta}$.

$\theta_z \to \theta_z^l$ and $\theta_P \to \theta_P^l$ is uniform on this set and therefore for every $\delta > 0$ there is an $\tilde{\sigma}$ such that $\theta_z^\star\left(x_\emptyset^\star + \delta\right) > \theta_P^\star\left(x_\emptyset^\star + \delta\right)$ and $\theta_z^\star\left(x_\emptyset^\star - \delta\right) < \theta_P^\star\left(x_\emptyset^\star - \delta\right)$. Taking $\delta \to 0$ establishes the result.

Consider a limiting opaque equilibrium such that $x_\emptyset^\star < \overline{\theta}$. By (17), we must have $\theta_\emptyset^\star = f(0)$ and therefore $\lim_{\tilde{\sigma}\to\infty} \Pr\left(\theta \le \theta_\emptyset^\star\right) = \Phi\left(\frac{f(0)-\overline{\theta}}{\hat{\sigma}}\right)$. Next, we compare the limiting probability of regime change under transparency. We show that all such equilibria involve a higher probability of regime change. Consider continuation equilibria satisfying $x_y^{\star,l} < \overline{\theta}$ wherever there are multiple equilibria (i.e. for all y such that $f(0) \le \theta_y^l(x) \le f(1)$). Then

$$\lim_{\tilde{\sigma}\to\infty} \Pr\left(\theta \le \theta_\emptyset^\star\right) = \Pr\left(y \ge y_1\right)\Phi\left(\frac{f(0)-\overline{\theta}}{\hat{\sigma}}\right) + \left(1 - \Pr\left(y \ge y_1\right)\right)\Phi\left(\frac{f(1)-\overline{\theta}}{\hat{\sigma}}\right) > \Phi\left(\frac{f(0)-\overline{\theta}}{\hat{\sigma}}\right)$$

where y_1 satisfies $\theta_{y_1}\left(\overline{\theta}\right) = f(1)$. A similar argument establishes the converse for $x_\emptyset^\star > \overline{\theta}$. □

Proof of Proposition 7

Proof. Comparison of the equilibrium threshold in κ is isomorphic to the comparative statics exercise in Proposition 4 of Iachan and Nenov [2015] for $U(\theta) = -c$, $D(\theta) = 1-c$. Moreover, since the equilibrium threshold under transparency does not depend on the realization of y, both the bandwagon and targeting effects must be 0. □

Proof of Proposition 8

Proof. There are four potential cases to analyze, depending on the position of the threshold 'pass' mark, $\hat{\theta}$: These are $\hat{\theta} \in (-\infty, f(0))$, $\hat{\theta} \in (f(0), \theta^*)$, $\hat{\theta} \in (\theta^*, f(1))$ and $\hat{\theta} \in (f(1), \infty)$. Suppose $\hat{\theta} \in (0, \theta^*)$.[53] We show here that then even the transparent equilibrium with the highest chance of regime change (if there are multiple equilibria) fails to increase the probability of a revolution, *ex ante*.

First, note that following a signal $y = 0$, $x_{y=0}^* = \infty$, $\theta_{y=0}^* = \hat{\theta}$ is a continuation equilibrium (since $\hat{\theta} \le \theta^* < f(1)$, there is only one-sided limit dominance), and must maximize the probability of regime change among all equilibria following $y = 0$.

Now consider continuation equilibrium following $y = 1$. The argument proceeds in two steps. First we consider only threshold equilibria. We then show that all other possible equilibria involve no greater probability of regime change.

Step 1: Consider any threshold equilibrium $\left(x_{y=1}^*, \theta_{y=1}^*\right)$ such that any agent with a private signal $x \le x_{y=1}^*$ will revolt and there is regime change in any state $\theta \le \theta_{y=1}^*$. Equilibrium

[53] The other three cases can be established by similar argument.

satisfies:

$$\theta' = f\left(\Phi\left(\frac{x' - \theta'}{\sigma_\epsilon}\right)\right) \tag{48}$$

$$\Pr\left(\theta \le \theta' \mid x', \theta > \hat{\theta}\right) = \frac{\Pr\left(\hat{\theta} < \theta < \theta' \mid x'\right)}{\Pr\left(\theta > \hat{\theta} \mid x'\right)} = c \tag{49}$$

Let $x_{y=1}(\theta')$ denote the largest x which solves (49) given θ'. Then

Lemma 11. $x_{y=1}(\theta') < x_\emptyset(\theta')\ \forall\theta'$.

Proof. $\frac{Pr\left(\hat{\theta}<\theta<\theta' \mid x\right)}{Pr\left(\theta>\hat{\theta} \mid x\right)} < Pr\left(\theta < \theta' \mid x\right)$ is a standard property of left-truncated distributions, for any $\hat{\theta} \in \mathbb{R}$. Now assume for a contradiction that $x_{y=1}(\theta') > x(\theta')$, for some $\theta' \in \Theta$. Since $x(\theta')$ uniquely solves (5) and $\Pr(\theta \le \theta' \mid x)$ is decreasing in x, it must be that $\Pr(\theta < \theta' \mid x_{y=1}(\theta')) < c$. But then it must also be the case that $\frac{\Pr\left(\hat{\theta}<\theta<\bar{\theta} \mid x_P\left(\bar{\theta}\right)\right)}{\Pr\left(\theta>\hat{\theta} \mid x_P\left(\bar{\theta}\right)\right)} < c$ - contradicting our assumption that $x_P\left(\bar{\theta}\right)$ solves (49). □

Now consider any solution $\left(x_{y=1}\left(\theta^*_{y=1}\right), \theta^*_{y=1}\right)$ to (48), (49). Let the set of the corresponding values of $\theta^*_{y=1}$ be $\Theta^*_{y=1}$. The following Lemma proves that any such cut-off equilibrium involves less coordination than the case without public information:

Lemma 12. $\sup_{\theta^*_{y=1} \in \Theta^*_{y=1}} \left\{\theta^*_{y=1}\right\} < \theta^*$.

Proof. Assume for a contradiction that $\exists\theta^*_{y=1}$ s.t. $\theta^*_{y=1} \ge \theta^*$. Since (i) (x^*, θ^*) uniquely solves (4)-(5) and (ii) by assumption (2), $\theta_P(x) - \theta_z(x)$ is decreasing, we must have $\theta - f\left(\Phi\left(\frac{x_\emptyset(\theta)-\theta}{\sigma_\epsilon}\right)\right) > 0, \forall\theta > \theta^*$. By Lemma 11, this means $\theta - f\left(\Phi\left(\frac{x_{y=1}(\theta)-\theta}{\sigma_\epsilon}\right)\right) > 0, \forall\theta > \theta^*$ - a contradiction to equilibrium at $\theta^*_{y=1}$. □

Step 2: We show that no other equilibrium involves a higher probability of regime change following $y = 1$ than the one we identified in step 1. This follows by straightforward application of the iteratred dominance argument in Angeletos et al. [2007].

Combined with $x^*_{y=0} = \infty$, $\theta^*_{y=0} = \hat{\theta}$, we have established the result. □

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